

Mathematica 11.3 Integration Test Results

Test results for the 3071 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 240: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x^3)^3 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^3)^4}{12 b}$$

Result (type 1, 43 leaves):

$$\frac{a^3 x^3}{3} + \frac{1}{2} a^2 b x^6 + \frac{1}{3} a b^2 x^9 + \frac{b^3 x^{12}}{12}$$

Problem 245: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^3}{x^{13}} dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{(a + b x^3)^4}{12 a x^{12}}$$

Result (type 1, 43 leaves):

$$-\frac{a^3}{12 x^{12}} - \frac{a^2 b}{3 x^9} - \frac{a b^2}{2 x^6} - \frac{b^3}{3 x^3}$$

Problem 262: Result more than twice size of optimal antiderivative.

$$\int x^5 (a + b x^3)^5 dx$$

Optimal (type 1, 34 leaves, 3 steps):

$$-\frac{a (a + b x^3)^6}{18 b^2} + \frac{(a + b x^3)^7}{21 b^2}$$

Result (type 1, 69 leaves):

$$\frac{a^5 x^6}{6} + \frac{5}{9} a^4 b x^9 + \frac{5}{6} a^3 b^2 x^{12} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{18} a b^4 x^{18} + \frac{b^5 x^{21}}{21}$$

Problem 263: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x^3)^5 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^3)^6}{18 b}$$

Result (type 1, 69 leaves):

$$\frac{a^5 x^3}{3} + \frac{5}{6} a^4 b x^6 + \frac{10}{9} a^3 b^2 x^9 + \frac{5}{6} a^2 b^3 x^{12} + \frac{1}{3} a b^4 x^{15} + \frac{b^5 x^{18}}{18}$$

Problem 270: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^5}{x^{19}} dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{(a + b x^3)^6}{18 a x^{18}}$$

Result (type 1, 69 leaves):

$$-\frac{a^5}{18 x^{18}} - \frac{a^4 b}{3 x^{15}} - \frac{5 a^3 b^2}{6 x^{12}} - \frac{10 a^2 b^3}{9 x^9} - \frac{5 a b^4}{6 x^6} - \frac{b^5}{3 x^3}$$

Problem 289: Result more than twice size of optimal antiderivative.

$$\int x^8 (a + b x^3)^8 dx$$

Optimal (type 1, 53 leaves, 3 steps):

$$\frac{a^2 (a + b x^3)^9}{27 b^3} - \frac{a (a + b x^3)^{10}}{15 b^3} + \frac{(a + b x^3)^{11}}{33 b^3}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^9}{9} + \frac{2}{3} a^7 b x^{12} + \frac{28}{15} a^6 b^2 x^{15} + \frac{28}{9} a^5 b^3 x^{18} + \frac{10}{3} a^4 b^4 x^{21} + \frac{7}{3} a^3 b^5 x^{24} + \frac{28}{27} a^2 b^6 x^{27} + \frac{4}{15} a b^7 x^{30} + \frac{b^8 x^{33}}{33}$$

Problem 290: Result more than twice size of optimal antiderivative.

$$\int x^5 (a + b x^3)^8 dx$$

Optimal (type 1, 34 leaves, 3 steps):

$$-\frac{a (a + b x^3)^9}{27 b^2} + \frac{(a + b x^3)^{10}}{30 b^2}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^6}{6} + \frac{8}{9} a^7 b x^9 + \frac{7}{3} a^6 b^2 x^{12} + \frac{56}{15} a^5 b^3 x^{15} + \frac{35}{9} a^4 b^4 x^{18} + \frac{8}{3} a^3 b^5 x^{21} + \frac{7}{6} a^2 b^6 x^{24} + \frac{8}{27} a b^7 x^{27} + \frac{b^8 x^{30}}{30}$$

Problem 291: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x^3)^8 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^3)^9}{27 b}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^3}{3} + \frac{4}{3} a^7 b x^6 + \frac{28}{9} a^6 b^2 x^9 + \frac{14}{3} a^5 b^3 x^{12} + \frac{14}{3} a^4 b^4 x^{15} + \frac{28}{9} a^3 b^5 x^{18} + \frac{4}{3} a^2 b^6 x^{21} + \frac{1}{3} a b^7 x^{24} + \frac{b^8 x^{27}}{27}$$

Problem 301: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^8}{x^{28}} dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{(a + b x^3)^9}{27 a x^{27}}$$

Result (type 1, 108 leaves):

$$-\frac{a^8}{27 x^{27}} - \frac{a^7 b}{3 x^{24}} - \frac{4 a^6 b^2}{3 x^{21}} - \frac{28 a^5 b^3}{9 x^{18}} - \frac{14 a^4 b^4}{3 x^{15}} - \frac{14 a^3 b^5}{3 x^{12}} - \frac{28 a^2 b^6}{9 x^9} - \frac{4 a b^7}{3 x^6} - \frac{b^8}{3 x^3}$$

Problem 302: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^8}{x^{31}} dx$$

Optimal (type 1, 40 leaves, 3 steps):

$$-\frac{(a + b x^3)^9}{30 a x^{30}} + \frac{b (a + b x^3)^9}{270 a^2 x^{27}}$$

Result (type 1, 108 leaves):

$$-\frac{a^8}{30 x^{30}} - \frac{8 a^7 b}{27 x^{27}} - \frac{7 a^6 b^2}{6 x^{24}} - \frac{8 a^5 b^3}{3 x^{21}} - \frac{35 a^4 b^4}{9 x^{18}} - \frac{56 a^3 b^5}{15 x^{15}} - \frac{7 a^2 b^6}{3 x^{12}} - \frac{8 a b^7}{9 x^9} - \frac{b^8}{6 x^6}$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+a-bx^3} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(1+a)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}(1+a)^{2/3}b^{1/3}} - \frac{\text{Log}\left[(1+a)^{1/3}-b^{1/3}x\right]}{3(1+a)^{2/3}b^{1/3}} + \frac{\text{Log}\left[(1+a)^{2/3}+(1+a)^{1/3}b^{1/3}x+b^{2/3}x^2\right]}{6(1+a)^{2/3}b^{1/3}}$$

Result (type 3, 124 leaves):

$$\frac{1}{6(1+a)^{2/3}b^{1/3}}(-1)^{2/3}\left(-2\sqrt{3}\text{ArcTan}\left[\frac{-1+\frac{2(-1)^{1/3}b^{1/3}x}{(1+a)^{1/3}}}{\sqrt{3}}\right]-\right. \\ \left.2\text{Log}\left[(1+a)^{1/3}+(-1)^{1/3}b^{1/3}x\right]+\text{Log}\left[(1+a)^{2/3}-(-1)^{1/3}(1+a)^{1/3}b^{1/3}x+(-1)^{2/3}b^{2/3}x^2\right]\right)$$

Problem 366: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{-1+a-bx^3} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2b^{1/3}x}{(1-a)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}(1-a)^{2/3}b^{1/3}} - \frac{\text{Log}\left[(1-a)^{1/3}+b^{1/3}x\right]}{3(1-a)^{2/3}b^{1/3}} + \frac{\text{Log}\left[(1-a)^{2/3}-(1-a)^{1/3}b^{1/3}x+b^{2/3}x^2\right]}{6(1-a)^{2/3}b^{1/3}}$$

Result (type 3, 124 leaves):

$$\frac{1}{6(-1+a)^{2/3}b^{1/3}}(-1)^{2/3}\left(-2\sqrt{3}\text{ArcTan}\left[\frac{-1+\frac{2(-1)^{1/3}b^{1/3}x}{(-1+a)^{1/3}}}{\sqrt{3}}\right]-\right. \\ \left.2\text{Log}\left[(-1+a)^{1/3}+(-1)^{1/3}b^{1/3}x\right]+\text{Log}\left[(-1+a)^{2/3}-(-1)^{1/3}(-1+a)^{1/3}b^{1/3}x+(-1)^{2/3}b^{2/3}x^2\right]\right)$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int x^6 \sqrt{a+bx^3} dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{48 a^2 x \sqrt{a+b x^3}}{935 b^2} + \frac{6 a x^4 \sqrt{a+b x^3}}{187 b} + \frac{2}{17} x^7 \sqrt{a+b x^3} + \\
 & \left(32 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^3 (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(935 b^{7/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 184 leaves):

$$\begin{aligned}
 & \sqrt{a+b x^3} \left(-\frac{48 a^2 x}{935 b^2} + \frac{6 a x^4}{187 b} + \frac{2 x^7}{17} \right) + \\
 & \left(32 i 3^{3/4} a^{10/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(935 (-b)^{1/3} b^2 \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \sqrt{a+b x^3} dx$$

Optimal (type 4, 251 leaves, 3 steps):

$$\begin{aligned}
 & \frac{6 a x \sqrt{a+b x^3}}{55 b} + \frac{2}{11} x^4 \sqrt{a+b x^3} - \left(4 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^2 (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(55 b^{4/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 168 leaves):

$$\frac{2 x \sqrt{a+b x^3} (3 a+5 b x^3)}{55 b} + \left(4 i 3^{3/4} a^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(55 (-b)^{4/3} \sqrt{a+b x^3} \right)$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b x^3} dx$$

Optimal (type 4, 227 leaves, 2 steps):

$$\frac{2}{5} x \sqrt{a+b x^3} + \left(2 \times 3^{3/4} \sqrt{2+\sqrt{3}} a (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(5 b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a+b x^3} \right)$$

Result (type 4, 155 leaves):

$$\frac{2}{5} x \sqrt{a+b x^3} + \left(2 i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(5 (-b)^{1/3} \sqrt{a+b x^3} \right)$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}}{x^3} dx$$

Optimal (type 4, 228 leaves, 2 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^3}}{2x^2} + \left(3^{3/4} \sqrt{2+\sqrt{3}} b^{2/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(2 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a+bx^3} \right)
 \end{aligned}$$

Result (type 4, 158 leaves):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^3}}{2x^2} + \left(i 3^{3/4} a^{1/3} b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
 & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(2 (-b)^{1/3} \sqrt{a+bx^3} \right)
 \end{aligned}$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx^3}}{x^6} dx$$

Optimal (type 4, 253 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^3}}{5x^5} - \frac{3b\sqrt{a+bx^3}}{20ax^2} - \\
 & \left(3^{3/4} \sqrt{2+\sqrt{3}} b^{5/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(20a \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a+bx^3} \right)
 \end{aligned}$$

Result (type 4, 173 leaves):

$$\left(-\frac{1}{5x^5} - \frac{3b}{20ax^2} \right) \sqrt{a+bx^3} - \left(i 3^{3/4} b^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(20 a^{2/3} (-b)^{1/3} \sqrt{a+bx^3} \right)$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx^3}}{x^9} dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$-\frac{\sqrt{a+bx^3}}{8x^8} - \frac{3b\sqrt{a+bx^3}}{80ax^5} + \frac{21b^2\sqrt{a+bx^3}}{320a^2x^2} + \\ \left(7 \times 3^{3/4} \sqrt{2+\sqrt{3}} b^{8/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right. \\ \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\ \left(320 a^2 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 181 leaves):

$$\left(-40 a^3 - 52 a^2 b x^3 + 9 a b^2 x^6 + 21 b^3 x^9 - \right. \\ \left. 7 i 3^{3/4} a^{1/3} (-b)^{8/3} x^8 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(320 a^2 x^8 \sqrt{a+bx^3} \right)$$

Problem 382: Result unnecessarily involves imaginary or complex numbers.

$$\int x^7 \sqrt{a + b x^3} \, dx$$

Optimal (type 4, 535 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{60 a^2 x^2 \sqrt{a + b x^3}}{1729 b^2} + \frac{6 a x^5 \sqrt{a + b x^3}}{247 b} + \frac{2}{19} x^8 \sqrt{a + b x^3} + \\
 & \frac{240 a^3 \sqrt{a + b x^3}}{1729 b^{8/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \left(120 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{10/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(80 \sqrt{2} 3^{3/4} a^{10/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)
 \end{aligned}$$

Result (type 4, 238 leaves):

$$\begin{aligned}
 & - \left(\left(2 \left((-b)^{2/3} (a + b x^3) (30 a^2 x^2 - 21 a b x^5 - 91 b^2 x^8) + \right. \right. \right. \\
 & \quad 40 (-1)^{2/3} 3^{3/4} a^{11/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \\
 & \quad \left. \left. \left(\sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{5/6} \right. \right. \right. \\
 & \quad \left. \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) \right) / \left(1729 (-b)^{8/3} \sqrt{a + b x^3} \right)
 \end{aligned}$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{a + b x^3} \, dx$$

Optimal (type 4, 511 leaves, 5 steps):

$$\begin{aligned}
 & \frac{6 a x^2 \sqrt{a+b x^3}}{91 b} + \frac{2}{13} x^5 \sqrt{a+b x^3} - \frac{24 a^2 \sqrt{a+b x^3}}{91 b^{5/3} \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \\
 & \left(12 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{7/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(91 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) - \left(8 \sqrt{2} 3^{3/4} a^{7/3} \left(a^{1/3} + b^{1/3} x \right) \right. \\
 & \quad \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(91 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result(type 4, 231 leaves):

$$\begin{aligned}
 & \frac{2 \sqrt{a+b x^3} \left(3 a x^2 + 7 b x^5 \right)}{91 b} + \\
 & \left(8 (-1)^{1/6} 3^{3/4} a^{8/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
 & \quad \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \\
 & \quad \left. \left. (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right] \right) / \left(91 (-b)^{5/3} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 384: Result unnecessarily involves imaginary or complex numbers.

$$\int x \sqrt{a+b x^3} dx$$

Optimal (type 4, 487 leaves, 4 steps):

$$\frac{2}{7} x^2 \sqrt{a + b x^3} + \frac{6 a \sqrt{a + b x^3}}{7 b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \left(3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\ \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(7 b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(2 \sqrt{2} 3^{3/4} a^{4/3} (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(7 b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 218 leaves):

$$\frac{2}{7} x^2 \sqrt{a + b x^3} + \left(2 (-1)^{1/6} 3^{3/4} a^{5/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \right. \\ \left. \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \right. \\ \left. \left. (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(7 (-b)^{2/3} \sqrt{a + b x^3} \right)$$

Problem 385: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + b x^3}}{x^2} dx$$

Optimal (type 4, 479 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^3}}{x} + \frac{3b^{1/3}\sqrt{a+bx^3}}{(1+\sqrt{3})a^{1/3}+b^{1/3}x} - \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{1/3} b^{1/3} (a^{1/3}+b^{1/3}x) \sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \right. \\
 & \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(2 \sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3} \right) + \left(\sqrt{2} 3^{3/4} a^{1/3} b^{1/3} (a^{1/3}+b^{1/3}x) \right. \\
 & \quad \left. \sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(\sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3} \right)
 \end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^3}}{x} + \left((-1)^{1/6} 3^{3/4} a^{2/3} b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3}x}{a^{1/3}} + \frac{(-b)^{2/3}x^2}{a^{2/3}}} \right. \\
 & \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
 & \quad \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \left. \right) / \left((-b)^{2/3} \sqrt{a+bx^3} \right)
 \end{aligned}$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx^3}}{x^5} dx$$

Optimal (type 4, 511 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b x^3}}{4 x^4} - \frac{3 b \sqrt{a+b x^3}}{8 a x} + \frac{3 b^{4/3} \sqrt{a+b x^3}}{8 a \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \left(16 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) + \left(3^{3/4} b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \left(4 \sqrt{2} a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 231 leaves):

$$\begin{aligned}
 & -\frac{\sqrt{a+b x^3} (2 a+3 b x^3)}{8 a x^4} + \frac{1}{8 a^{1/3} \sqrt{a+b x^3}} (-1)^{1/6} 3^{3/4} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \\
 & \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \\
 & \left. (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)
 \end{aligned}$$

Problem 394: Result unnecessarily involves imaginary or complex numbers.

$$\int x^6 (a+b x^3)^{3/2} dx$$

Optimal (type 4, 296 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{432 a^3 x \sqrt{a+b x^3}}{21505 b^2} + \frac{54 a^2 x^4 \sqrt{a+b x^3}}{4301 b} + \frac{18}{391} a x^7 \sqrt{a+b x^3} + \\
 & \frac{2}{23} x^7 (a+b x^3)^{3/2} + \left(288 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^4 (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(21505 b^{7/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 195 leaves):

$$\begin{aligned}
 & \sqrt{a+b x^3} \left(-\frac{432 a^3 x}{21505 b^2} + \frac{54 a^2 x^4}{4301 b} + \frac{52 a x^7}{391} + \frac{2 b x^{10}}{23} \right) + \\
 & \left(288 i 3^{3/4} a^{13/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
 & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(21505 (-b)^{1/3} b^2 \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 (a+b x^3)^{3/2} dx$$

Optimal (type 4, 272 leaves, 4 steps):

$$\frac{54 a^2 x \sqrt{a + b x^3}}{935 b} + \frac{18}{187} a x^4 \sqrt{a + b x^3} + \frac{2}{17} x^4 (a + b x^3)^{3/2} -$$

$$\left(36 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^3 (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) /$$

$$\left(935 b^{4/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 178 leaves):

$$- \left(\left(2 \left((-b)^{1/3} (a + b x^3) (27 a^2 x + 100 a b x^4 + 55 b^2 x^7) - \right. \right. \right.$$

$$18 i 3^{3/4} a^{10/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left. \right)$$

$$\left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right) / \left(935 (-b)^{4/3} \sqrt{a + b x^3} \right)$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + b x^3)^{3/2} dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{18}{55} a x \sqrt{a + b x^3} + \frac{2}{11} x (a + b x^3)^{3/2} +$$

$$\left(18 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right/$$

$$\left(55 b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 166 leaves):

$$\sqrt{a + b x^3} \left(\frac{28 a x}{55} + \frac{2 b x^4}{11} \right) +$$

$$\left(18 i 3^{3/4} a^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i (-b)^{1/3} x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right/ \left(55 (-b)^{1/3} \sqrt{a + b x^3} \right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^3)^{3/2}}{x^3} dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{9}{10} b x \sqrt{a + b x^3} - \frac{(a + b x^3)^{3/2}}{2 x^2} +$$

$$\left(9 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a b^{2/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \text{EllipticF}\left[\right.$$

$$\left. \text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right/ \left(10 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 167 leaves):

$$\left(-\frac{a}{2x^2} + \frac{2bx}{5}\right) \sqrt{a+bx^3} + \left(9i3^{3/4}a^{4/3}b\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right. \\ \left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) / \left(10(-b)^{1/3}\sqrt{a+bx^3}\right)$$

Problem 398: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+bx^3)^{3/2}}{x^6} dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{9b\sqrt{a+bx^3}}{20x^2} - \frac{(a+bx^3)^{3/2}}{5x^5} + \left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}} b^{5/3} (a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \text{EllipticF}\left[\right. \right. \\ \left. \left. \text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x}\right], -7-4\sqrt{3}\right]\right) / \left(20 \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a+bx^3}\right)$$

Result (type 4, 167 leaves):

$$-\frac{\sqrt{a+bx^3}(4a+13bx^3)}{20x^5} + \frac{1}{20\sqrt{a+bx^3}} 9i3^{3/4}a^{1/3}(-b)^{5/3} \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)} \\ \sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 399: Result unnecessarily involves imaginary or complex numbers.

$$\int x^7 (a+bx^3)^{3/2} dx$$

Optimal (type 4, 556 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{108 a^3 x^2 \sqrt{a+b x^3}}{8645 b^2} + \frac{54 a^2 x^5 \sqrt{a+b x^3}}{6175 b} + \frac{18}{475} a x^8 \sqrt{a+b x^3} + \\
 & \frac{432 a^4 \sqrt{a+b x^3}}{8645 b^{8/3} \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \frac{2}{25} x^8 (a+b x^3)^{3/2} - \left(216 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{13/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(8645 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) + \left(144 \sqrt{2} 3^{3/4} a^{13/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(8645 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 253 leaves):

$$\begin{aligned}
 & \frac{2 x^2 \sqrt{a+b x^3} (-270 a^3 + 189 a^2 b x^3 + 2548 a b^2 x^6 + 1729 b^3 x^9)}{43225 b^2} + \\
 & \left(144 (-1)^{1/6} 3^{3/4} a^{14/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
 & \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}}\right], (-1)^{1/3}\right] + \right. \\
 & \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) \left. \right) / \left(8645 (-b)^{8/3} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 400: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (a+b x^3)^{3/2} dx$$

Optimal (type 4, 532 leaves, 6 steps):

$$\frac{54 a^2 x^2 \sqrt{a+b x^3}}{1729 b} + \frac{18}{247} a x^5 \sqrt{a+b x^3} - \frac{216 a^3 \sqrt{a+b x^3}}{1729 b^{5/3} \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} +$$

$$\frac{2}{19} x^5 (a+b x^3)^{3/2} + \left(108 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{10/3} (a^{1/3} + b^{1/3} x) \right.$$

$$\left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) - \left(72 \sqrt{2} 3^{3/4} a^{10/3} (a^{1/3} + b^{1/3} x) \right.$$

$$\left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)$$

Result (type 4, 238 leaves):

$$- \left(\left(\left(\left((-b)^{2/3} (a+b x^3) (27 a^2 x^2 + 154 a b x^5 + 91 b^2 x^8) + \right. \right. \right. \right.$$

$$36 (-1)^{2/3} 3^{3/4} a^{11/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left. \left. \left. \left. \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{5/6} \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right] \right) \right) \right) / \left(1729 (-b)^{5/3} \sqrt{a+b x^3} \right)$$

Problem 401: Result unnecessarily involves imaginary or complex numbers.

$$\int x (a + b x^3)^{3/2} dx$$

Optimal (type 4, 508 leaves, 5 steps):

$$\begin{aligned} & \frac{18}{91} a x^2 \sqrt{a + b x^3} + \frac{54 a^2 \sqrt{a + b x^3}}{91 b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \\ & \frac{2}{13} x^2 (a + b x^3)^{3/2} - \left(27 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} (a^{1/3} + b^{1/3} x) \right. \\ & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(91 b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(18 \sqrt{2} 3^{3/4} a^{7/3} (a^{1/3} + b^{1/3} x) \right. \\ & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(91 b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) \end{aligned}$$

Result (type 4, 229 leaves):

$$\sqrt{a + b x^3} \left(\frac{32 a x^2}{91} + \frac{2 b x^5}{13} \right) +$$

$$\left(18 (-1)^{1/6} 3^{3/4} a^{8/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. - i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right.$$

$$\left. (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(91 (-b)^{2/3} \sqrt{a + b x^3} \right)$$

Problem 402: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^3)^{3/2}}{x^2} dx$$

Optimal (type 4, 504 leaves, 5 steps):

$$\frac{9}{7} b x^2 \sqrt{a + b x^3} + \frac{27 a b^{1/3} \sqrt{a + b x^3}}{7 \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \frac{(a + b x^3)^{3/2}}{x}$$

$$\left(27 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} b^{1/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(14 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(9 \sqrt{2} 3^{3/4} a^{4/3} b^{1/3} (a^{1/3} + b^{1/3} x) \right.$$

$$\left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result(type 4, 228 leaves):

$$\left(-\frac{a}{x} + \frac{2 b x^2}{7} \right) \sqrt{a + b x^3} +$$

$$\left(9 (-1)^{1/6} 3^{3/4} a^{5/3} b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right.$$

$$\left. \left. (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(7 (-b)^{2/3} \sqrt{a + b x^3} \right)$$

Problem 403: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^3)^{3/2}}{x^5} dx$$

Optimal (type 4, 505 leaves, 5 steps):

$$-\frac{9 b \sqrt{a + b x^3}}{8 x} + \frac{27 b^{4/3} \sqrt{a + b x^3}}{8 \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \frac{(a + b x^3)^{3/2}}{4 x^4} - \left(27 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{1/3} b^{4/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \left(16 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(9 \times 3^{3/4} a^{1/3} b^{4/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \left(4 \sqrt{2} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 228 leaves):

$$-\frac{\sqrt{a + b x^3} (2 a + 11 b x^3)}{8 x^4} + \frac{1}{8 \sqrt{a + b x^3}} 9 (-1)^{1/6} 3^{3/4} a^{2/3} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(-i \sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 411: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a + b x^3}} dx$$

Optimal (type 4, 254 leaves, 3 steps):

$$-\frac{16 a x \sqrt{a + b x^3}}{55 b^2} + \frac{2 x^4 \sqrt{a + b x^3}}{11 b} + \left(32 \sqrt{2 + \sqrt{3}} a^2 (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) / \left(55 \times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 174 leaves):

$$\sqrt{a + b x^3} \left(-\frac{16 a x}{55 b^2} + \frac{2 x^4}{11 b} \right) + \left(32 i a^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(55 \times 3^{1/4} (-b)^{1/3} b^2 \sqrt{a + b x^3} \right)$$

Problem 412: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{a + b x^3}} dx$$

Optimal (type 4, 230 leaves, 2 steps):

$$\frac{2 x \sqrt{a+b x^3}}{5 b} - \left(4 \sqrt{2+\sqrt{3}} a \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right] \right) / \left(5 \times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3} \right)$$

Result (type 4, 158 leaves):

$$\frac{2 x \sqrt{a+b x^3}}{5 b} + \left(4 i a^{4/3} \sqrt{\left(-1\right)^{5/6} \left(-1+\frac{\left(-b\right)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{\left(-b\right)^{1/3} x}{a^{1/3}}+\frac{\left(-b\right)^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\left(-b\right)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right] \right) / \left(5 \times 3^{1/4} \left(-b\right)^{4/3} \sqrt{a+b x^3} \right)$$

Problem 413: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b x^3}} dx$$

Optimal (type 4, 207 leaves, 1 step):

$$\left(2 \sqrt{2+\sqrt{3}} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right] \right) / \left(3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3} \right)$$

Result (type 4, 136 leaves):

$$\left(2 i a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(3^{1/4} (-b)^{1/3} \sqrt{a + b x^3} \right)$$

Problem 414: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{a + b x^3}} dx$$

Optimal (type 4, 234 leaves, 2 steps):

$$-\frac{\sqrt{a + b x^3}}{2 a x^2} - \left(\sqrt{2 + \sqrt{3}} b^{2/3} (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) / \\ \left(2 \times 3^{1/4} a \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 161 leaves):

$$-\frac{\sqrt{a + b x^3}}{2 a x^2} - \left(i b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(2 \times 3^{1/4} a^{2/3} (-b)^{1/3} \sqrt{a + b x^3} \right)$$

Problem 415: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{a + b x^3}} dx$$

Optimal (type 4, 256 leaves, 3 steps):

$$-\frac{\sqrt{a+bx^3}}{5ax^5} + \frac{7b\sqrt{a+bx^3}}{20a^2x^2} + \left(7\sqrt{2+\sqrt{3}}b^{5/3}(a^{1/3}+b^{1/3}x) \right. \\ \left. \sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(20 \times 3^{1/4} a^2 \sqrt{\frac{a^{1/3}(a^{1/3}+b^{1/3}x)}{((1+\sqrt{3})a^{1/3}+b^{1/3}x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 170 leaves):

$$\frac{1}{60a^2x^5\sqrt{a+bx^3}} \left(-12a^2 + 9abx^3 + 21b^2x^6 + 7i3^{3/4}a^{1/3}(-b)^{5/3}x^5 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}x}{a^{1/3}} \right)} \right. \\ \left. \sqrt{1 + \frac{(-b)^{1/3}x}{a^{1/3}} + \frac{(-b)^{2/3}x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 416: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{a+bx^3}} dx$$

Optimal (type 4, 514 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{20 a x^2 \sqrt{a+b x^3}}{91 b^2} + \frac{2 x^5 \sqrt{a+b x^3}}{13 b} + \frac{80 a^2 \sqrt{a+b x^3}}{91 b^{8/3} \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \\
 & \left(40 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{7/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(91 b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) + \left(80 \sqrt{2} a^{7/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(91 \times 3^{1/4} b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result(type 4, 228 leaves):

$$\begin{aligned}
 & - \left(\left(2 \left(3 (-b)^{2/3} (a+b x^3) (10 a x^2 - 7 b x^5) + \right. \right. \right. \\
 & 40 (-1)^{2/3} 3^{3/4} a^{8/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \\
 & \left. \left. \left(\sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{5/6} \right. \right. \right. \\
 & \left. \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) \right) / \left(273 (-b)^{8/3} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 417: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + b x^3}} dx$$

Optimal (type 4, 490 leaves, 4 steps):

$$\frac{2 x^2 \sqrt{a + b x^3}}{7 b} - \frac{8 a \sqrt{a + b x^3}}{7 b^{5/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} +$$

$$\left(4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) - \left(8 \sqrt{2} a^{4/3} \left(a^{1/3} + b^{1/3} x \right) \right.$$

$$\left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 \times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 221 leaves):

$$\frac{2 x^2 \sqrt{a + b x^3}}{7 b} + \left(8 (-1)^{1/6} a^{5/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \right. \right.$$

$$\left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(7 \times 3^{1/4} (-b)^{5/3} \sqrt{a + b x^3} \right)$$

Problem 418: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a + b x^3}} dx$$

Optimal (type 4, 462 leaves, 3 steps):

$$\frac{2 \sqrt{a + b x^3}}{b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \left(3^{1/4} \sqrt{2 - \sqrt{3}} a^{1/3} (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(2 \sqrt{2} a^{1/3} (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 197 leaves):

$$\left(2 (-1)^{1/6} a^{2/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left(-i \sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \\ \left. (-1)^{1/3} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \left. \right) / \left(3^{1/4} (-b)^{2/3} \sqrt{a + b x^3} \right)$$

Problem 419: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + b x^3}} dx$$

Optimal (type 4, 484 leaves, 4 steps):

$$-\frac{\sqrt{a + b x^3}}{a x} + \frac{b^{1/3} \sqrt{a + b x^3}}{a \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \left(3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\ \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(2 a^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) + \left(\sqrt{2} b^{1/3} (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(3^{1/4} a^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 217 leaves):

$$-\frac{\sqrt{a + b x^3}}{a x} + \left((-1)^{1/6} b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \right. \\ \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(3^{1/4} a^{1/3} (-b)^{2/3} \sqrt{a + b x^3} \right)$$

Problem 420: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{a + b x^3}} dx$$

Optimal (type 4, 514 leaves, 5 steps):

$$\begin{aligned} & -\frac{\sqrt{a + b x^3}}{4 a x^4} + \frac{5 b \sqrt{a + b x^3}}{8 a^2 x} - \frac{5 b^{4/3} \sqrt{a + b x^3}}{8 a^2 \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \\ & \left(5 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\ & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(16 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) - \left(5 b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \right. \\ & \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(4 \sqrt{2} 3^{1/4} a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) \end{aligned}$$

Result (type 4, 231 leaves):

$$\frac{\sqrt{a + b x^3} (-2 a + 5 b x^3)}{8 a^2 x^4} - \left(5 (-1)^{1/6} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. - i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\ \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(8 \times 3^{1/4} a^{4/3} \sqrt{a + b x^3} \right)$$

Problem 428: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(a + b x^3)^{3/2}} dx$$

Optimal (type 4, 251 leaves, 3 steps):

$$-\frac{2 x^4}{3 b \sqrt{a + b x^3}} + \frac{16 x \sqrt{a + b x^3}}{15 b^2} - \left(32 \sqrt{2 + \sqrt{3}} a (a^{1/3} + b^{1/3} x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right] \right) / \\ \left(15 \times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 161 leaves):

$$\left(6 (-b)^{1/3} x (8a + 3bx^3) - 32 i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(45 (-b)^{7/3} \sqrt{a + bx^3}\right)$$

Problem 429: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{(a + bx^3)^{3/2}} dx$$

Optimal (type 4, 229 leaves, 2 steps):

$$-\frac{2x}{3b\sqrt{a+bx^3}} + \left(4\sqrt{2+\sqrt{3}} (a^{1/3} + b^{1/3}x) \right. \\ \left. \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(3 \times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a + bx^3} \right)$$

Result (type 4, 151 leaves):

$$\left(6 (-b)^{1/3} x - 4 i 3^{3/4} a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(9 (-b)^{4/3} \sqrt{a + bx^3}\right)$$

Problem 430: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + bx^3)^{3/2}} dx$$

Optimal (type 4, 232 leaves, 2 steps):

$$\frac{2x}{3a\sqrt{a+bx^3}} + \left(2\sqrt{2+\sqrt{3}} (a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \left(3 \times 3^{1/4} a b^{1/3} \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 154 leaves):

$$\left(6(-b)^{1/3}x + 2i3^{3/4}a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3}x}{a^{1/3}} + \frac{(-b)^{2/3}x^2}{a^{2/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(9a(-b)^{1/3} \sqrt{a+bx^3} \right)$$

Problem 431: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}} dx$$

Optimal (type 4, 255 leaves, 3 steps):

$$\frac{2}{3ax^2\sqrt{a+bx^3}} - \frac{7\sqrt{a+bx^3}}{6a^2x^2} - \left(7\sqrt{2+\sqrt{3}} b^{2/3} (a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \left(6 \times 3^{1/4} a^2 \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 170 leaves):

$$\left(-3 (-b)^{1/3} (3a + 7bx^3) - 7i 3^{3/4} a^{1/3} bx^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(18a^2 (-b)^{1/3} x^2 \sqrt{a + bx^3}\right)$$

Problem 432: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 (a + bx^3)^{3/2}} dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$\frac{2}{3ax^5 \sqrt{a + bx^3}} - \frac{13\sqrt{a + bx^3}}{15a^2 x^5} + \frac{91b\sqrt{a + bx^3}}{60a^3 x^2} + \\ \left(91\sqrt{2 + \sqrt{3}} b^{5/3} (a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + b^{1/3}x}{(1 + \sqrt{3})a^{1/3} + b^{1/3}x}\right], -7 - 4\sqrt{3}\right] \right) / \\ \left(60 \times 3^{1/4} a^3 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3}x)}{((1 + \sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a + bx^3} \right)$$

Result (type 4, 183 leaves):

$$\left(3 (-b)^{1/3} (-12a^2 + 39abx^3 + 91b^2x^6) + \right. \\ \left. 91i 3^{3/4} a^{1/3} b^2 x^5 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(180a^3 (-b)^{1/3} x^5 \sqrt{a + bx^3}\right)$$

Problem 433: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{(a + b x^3)^{3/2}} dx$$

Optimal (type 4, 511 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{2 x^5}{3 b \sqrt{a + b x^3}} + \frac{20 x^2 \sqrt{a + b x^3}}{21 b^2} - \frac{80 a \sqrt{a + b x^3}}{21 b^{8/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \\
 & \left(40 \sqrt{2 - \sqrt{3}} a^{4/3} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(7 \times 3^{3/4} b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) - \left(80 \sqrt{2} a^{4/3} (a^{1/3} + b^{1/3} x) \right. \\
 & \quad \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(21 \times 3^{1/4} b^{8/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)
 \end{aligned}$$

Result (type 4, 221 leaves):

$$\left(2 \left(3 (-b)^{2/3} x^2 (10 a + 3 b x^3) + \right. \right. \\ 40 (-1)^{2/3} 3^{3/4} a^{5/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \\ \left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{5/6} \right. \\ \left. \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right) \right) \right) / \left(63 (-b)^{8/3} \sqrt{a + b x^3} \right)$$

Problem 434: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(a + b x^3)^{3/2}} dx$$

Optimal (type 4, 487 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{2x^2}{3b\sqrt{a+bx^3}} + \frac{8\sqrt{a+bx^3}}{3b^{5/3}\left((1+\sqrt{3})a^{1/3}+b^{1/3}x\right)} - \\
 & \left(4\sqrt{2-\sqrt{3}}a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left((1+\sqrt{3})a^{1/3}+b^{1/3}x\right)^2}} \right. \\
 & \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(3^{3/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left((1+\sqrt{3})a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3} \right) + \left(8\sqrt{2}a^{1/3}\left(a^{1/3}+b^{1/3}x\right) \right. \\
 & \quad \left. \sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left((1+\sqrt{3})a^{1/3}+b^{1/3}x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3}+b^{1/3}x}{(1+\sqrt{3})a^{1/3}+b^{1/3}x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(3 \times 3^{1/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left((1+\sqrt{3})a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3} \right)
 \end{aligned}$$

Result(type 4, 216 leaves):

$$\begin{aligned}
 & \frac{1}{9b\sqrt{a+bx^3}} \\
 & 2 \left(-3x^2 + \frac{1}{(-b)^{2/3}} 4(-1)^{1/6} 3^{3/4} a^{2/3} \sqrt{\frac{(-1)^{5/6}(-a^{1/3}+(-b)^{1/3}x)}{a^{1/3}}} \sqrt{1 + \frac{(-b)^{1/3}x}{a^{1/3}} + \frac{(-b)^{2/3}x^2}{a^{2/3}}} \right. \\
 & \quad \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
 & \quad \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 435: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(a + b x^3)^{3/2}} dx$$

Optimal (type 4, 489 leaves, 4 steps):

$$\frac{2 x^2}{3 a \sqrt{a + b x^3}} - \frac{2 \sqrt{a + b x^3}}{3 a b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)} +$$

$$\left(\sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(3^{3/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) - \left(2 \sqrt{2} (a^{1/3} + b^{1/3} x) \right.$$

$$\left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(3 \times 3^{1/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)$$

Result (type 4, 212 leaves):

$$\frac{1}{9 a \sqrt{a + b x^3}}$$

$$2 \left(3 x^2 + \frac{1}{(-b)^{2/3}} (-1)^{2/3} 3^{3/4} a^{2/3} \sqrt{\frac{(-1)^{5/6} (-a^{1/3} + (-b)^{1/3} x)}{a^{1/3}}} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left(\sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right.$$

$$\left. \left. (-1)^{5/6} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right)$$

Problem 436: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (a + b x^3)^{3/2}} dx$$

Optimal (type 4, 513 leaves, 5 steps):

$$\begin{aligned}
 & \frac{2}{3 a x \sqrt{a+b x^3}} - \frac{5 \sqrt{a+b x^3}}{3 a^2 x} + \frac{5 b^{1/3} \sqrt{a+b x^3}}{3 a^2 \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} - \\
 & \left(5 \sqrt{2-\sqrt{3}} b^{1/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(2 \times 3^{3/4} a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) + \left(5 \sqrt{2} b^{1/3} \left(a^{1/3} + b^{1/3} x \right) \right. \\
 & \quad \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(3 \times 3^{1/4} a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)
 \end{aligned}$$

Result (type 4, 226 leaves):

$$\begin{aligned}
 & \left(-3 (-b)^{2/3} (3 a + 5 b x^3) - 5 (-1)^{2/3} 3^{3/4} a^{2/3} b x \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \right. \\
 & \quad \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(\sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \\
 & \quad \left. \left. (-1)^{5/6} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(9 a^2 (-b)^{2/3} x \sqrt{a+b x^3} \right)
 \end{aligned}$$

Problem 437: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 (a+b x^3)^{3/2}} dx$$

Optimal (type 4, 535 leaves, 6 steps):

$$\frac{2}{3 a x^4 \sqrt{a+b x^3}} - \frac{11 \sqrt{a+b x^3}}{12 a^2 x^4} + \frac{55 b \sqrt{a+b x^3}}{24 a^3 x} - \frac{55 b^{4/3} \sqrt{a+b x^3}}{24 a^3 \left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)} + \left(55 \sqrt{2-\sqrt{3}} b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4 \sqrt{3}\right] \right) / \left(16 \times 3^{3/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right) - \left(55 b^{4/3} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4 \sqrt{3}\right] \right) / \left(12 \sqrt{2} 3^{1/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a+b x^3} \right)$$

Result (type 4, 241 leaves):

$$\left(3 (-b)^{2/3} \left(-6 a^2 + 33 a b x^3 + 55 b^2 x^6 \right) + 55 (-1)^{2/3} 3^{3/4} a^{2/3} b^2 x^4 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right) \left(\sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}}\right], (-1)^{1/3}\right] + (-1)^{5/6} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) / \left(72 a^3 (-b)^{2/3} x^4 \sqrt{a+b x^3} \right)$$

Problem 446: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 136 leaves, 3 steps):

$$-\frac{16}{55} x \sqrt{1+x^3} + \frac{2}{11} x^4 \sqrt{1+x^3} + \left(32 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \left(55 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 108 leaves):

$$\frac{1}{165 \sqrt{1+x^3}} + 2 \left(3x(-8-3x^3+5x^6) + 16(-1)^{1/6} 3^{3/4} \sqrt{-(-1)^{1/6}((-1)^{2/3}+x)} \sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 447: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 120 leaves, 2 steps):

$$\frac{2}{5} x \sqrt{1+x^3} - \left(4 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \left(5 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 100 leaves):

$$\frac{1}{15 \sqrt{1+x^3}} \left(6 (x+x^4) - 4 (-1)^{1/6} 3^{3/4} \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \right. \\ \left. \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 448: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 103 leaves, 1 step):

$$\left(2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 88 leaves):

$$\frac{1}{3^{1/4} \sqrt{1+x^3}} 2 (-1)^{1/6} \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \\ \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 449: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{1+x^3}} dx$$

Optimal (type 4, 122 leaves, 2 steps):

$$-\frac{\sqrt{1+x^3}}{2x^2} - \left(\sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(2 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 104 leaves):

$$-\frac{1}{6x^2\sqrt{1+x^3}} \left(3 + 3x^3 + (-1)^{1/6} 3^{3/4} x^2 \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \right. \\ \left. \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 450: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{1+x^3}} dx$$

Optimal (type 4, 138 leaves, 3 steps):

$$-\frac{\sqrt{1+x^3}}{5x^5} + \frac{7\sqrt{1+x^3}}{20x^2} + \\ \left(7\sqrt{2+\sqrt{3}}(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right] \right) / \\ \left(20 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 110 leaves):

$$\frac{1}{60x^5\sqrt{1+x^3}} \\ \left(-12 + 9x^3 + 21x^6 + 7(-1)^{1/6} 3^{3/4} x^5 \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \right. \\ \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 451: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 262 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{20}{91} x^2 \sqrt{1+x^3} + \frac{2}{13} x^5 \sqrt{1+x^3} + \frac{80 \sqrt{1+x^3}}{91 (1+\sqrt{3}+x)} - \\
 & \left(40 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(91 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) + \\
 & \frac{80 \sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{91 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}
 \end{aligned}$$

Result (type 4, 145 leaves):

$$\begin{aligned}
 & \frac{1}{273 \sqrt{1+x^3}} \\
 & 2 \left(3 x^2 (1+x^3) (-10+7 x^3) - 40 \times 3^{3/4} \sqrt{-(-1)^{1/6} ((-1)^{2/3}+x)} \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2} \right. \\
 & \left. \left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\
 & \left. \left. (-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 452: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 246 leaves, 4 steps):

$$\frac{2}{7} x^2 \sqrt{1+x^3} - \frac{8 \sqrt{1+x^3}}{7(1+\sqrt{3}+x)} +$$

$$\left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(7 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) -$$

$$\frac{8\sqrt{2}(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{7 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 138 leaves):

$$\frac{1}{21 \sqrt{1+x^3}} 2 \left(3 x^2 (1+x^3) + 4 \times 3^{3/4} \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \right.$$

$$\left. \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left. (-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 453: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 224 leaves, 3 steps):

$$\frac{2 \sqrt{1+x^3}}{1+\sqrt{3}+x} -$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) + \frac{2\sqrt{2}(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 123 leaves):

$$-\frac{1}{3^{1/4} \sqrt{1+x^3}} 2 \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x\right) \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2}}$$

$$\left(\sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. (-1)^{5/6} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 454: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{1+x^3}} dx$$

Optimal (type 4, 238 leaves, 4 steps):

$$-\frac{\sqrt{1+x^3}}{x} + \frac{\sqrt{1+x^3}}{1+\sqrt{3}+x} -$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(2 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) + \frac{\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 138 leaves):

$$\frac{1}{3 \sqrt{1+x^3}} \left(-\frac{3(1+x^3)}{x} - 3^{3/4} \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x\right)} \right.$$

$$\left. \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \left(\sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left. (-1)^{5/6} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 455: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{1+x^3}} dx$$

Optimal (type 4, 262 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1+x^3}}{4x^4} + \frac{5\sqrt{1+x^3}}{8x} - \frac{5\sqrt{1+x^3}}{8(1+\sqrt{3}+x)} + \\
 & \left(5 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(16 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) - \frac{5(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{4\sqrt{2} 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}
 \end{aligned}$$

Result (type 4, 145 leaves):

$$\begin{aligned}
 & \frac{1}{24\sqrt{1+x^3}} \left(\frac{3(1+x^3)(-2+5x^3)}{x^4} + 5 \times 3^{3/4} \sqrt{-(-1)^{1/6}((-1)^{2/3}+x)} \right. \\
 & \left. \sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2} \left(\sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\
 & \left. \left. (-1)^{5/6} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 464: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 152 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{16}{55}x\sqrt{1-x^3} - \frac{2}{11}x^4\sqrt{1-x^3} - \\
 & \left(32\sqrt{2+\sqrt{3}}(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(55 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)
 \end{aligned}$$

Result (type 4, 93 leaves):

$$\frac{1}{165 \sqrt{1-x^3}} 2 \left(3 x (-8 + 3 x^3 + 5 x^6) + \right. \\ \left. 16 i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 465: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 134 leaves, 2 steps):

$$-\frac{2}{5} x \sqrt{1-x^3} - \\ \left(4 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(5 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)$$

Result (type 4, 86 leaves):

$$\frac{1}{15 \sqrt{1-x^3}} 2 \left(3 x (-1+x^3) + \right. \\ \left. 2 i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 466: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 115 leaves, 1 step):

$$- \left(\left(2 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \right. \\ \left. \left(3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) \right)$$

Result (type 4, 73 leaves):

$$\frac{1}{3^{1/4} \sqrt{1-x^3}} 2 i \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 467: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{1-x^3}} dx$$

Optimal (type 4, 136 leaves, 2 steps):

$$\begin{aligned} & -\frac{\sqrt{1-x^3}}{2x^2} - \left(\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\ & \left(2 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) \end{aligned}$$

Result (type 4, 90 leaves):

$$\begin{aligned} & \frac{1}{6x^2 \sqrt{1-x^3}} \left(-3 + 3x^3 + \right. \\ & \left. i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} x^2 \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \end{aligned}$$

Problem 468: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{1-x^3}} dx$$

Optimal (type 4, 154 leaves, 3 steps):

$$\begin{aligned} & -\frac{\sqrt{1-x^3}}{5x^5} - \frac{7\sqrt{1-x^3}}{20x^2} - \\ & \left(7 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\ & \left(20 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) \end{aligned}$$

Result (type 4, 95 leaves):

$$\frac{1}{60 x^5 \sqrt{1-x^3}} \left(-12 - 9 x^3 + 21 x^6 + \right. \\ \left. 7 i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} x^5 \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 469: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{80 \sqrt{1-x^3}}{91 (1+\sqrt{3}-x)} - \frac{20}{91} x^2 \sqrt{1-x^3} - \frac{2}{13} x^5 \sqrt{1-x^3} - \\ \left(40 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\ \left(91 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) + \\ \frac{80 \sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{91 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 144 leaves):

$$\frac{1}{273 \sqrt{1-x^3}} 2 \left(3 x^2 (-1+x^3) (10+7 x^3) + 40 (-1)^{1/6} 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} \right. \\ \left. \sqrt{1+x+x^2} \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\ \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 470: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 276 leaves, 4 steps):

$$\frac{8\sqrt{1-x^3}}{7(1+\sqrt{3-x})} - \frac{2}{7}x^2\sqrt{1-x^3} -$$

$$\left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(7 \sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3} \right) +$$

$$\frac{8\sqrt{2}(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right]}{7 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3}}$$

Result (type 4, 137 leaves):

$$\frac{1}{21\sqrt{1-x^3}} 2 \left(3x^2(-1+x^3) + 4(-1)^{1/6} 3^{3/4} \sqrt{(-1)^{5/6}(-1+x)} \right.$$

$$\left. \sqrt{1+x+x^2} \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 471: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{1-x^3}} dx$$

Optimal (type 4, 252 leaves, 3 steps):

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3-x}} - \left(3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(\sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3} \right) + \frac{2\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3}}$$

Result (type 4, 122 leaves):

$$\frac{1}{3^{1/4} \sqrt{1-x^3}} 2 (-1)^{1/6} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2}$$

$$\left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 472: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{1-x^3}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$-\frac{\sqrt{1-x^3}}{1+\sqrt{3-x}} - \frac{\sqrt{1-x^3}}{x} + \left(3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right] \right) /$$

$$\left(2 \sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3} \right) - \frac{\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3-x})^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3-x}}{1+\sqrt{3-x}}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3-x})^2}} \sqrt{1-x^3}}$$

Result (type 4, 133 leaves):

$$\frac{1}{3\sqrt{1-x^3}} \left(\frac{3(-1+x^3)}{x} + (-1)^{2/3} 3^{3/4} \sqrt{(-1)^{5/6}(-1+x)} \right. \\ \left. \sqrt{1+x+x^2} \left(\sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \right. \\ \left. \left. (-1)^{5/6} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right)$$

Problem 473: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{1-x^3}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$-\frac{5\sqrt{1-x^3}}{8(1+\sqrt{3}-x)} - \frac{\sqrt{1-x^3}}{4x^4} - \frac{5\sqrt{1-x^3}}{8x} + \\ \left(5 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right] \right) / \\ \left(16 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) - \frac{5(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{4\sqrt{2} 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 145 leaves):

$$\frac{1}{24x^4 \sqrt{1-x^3}} \left(3(-1+x^3)(2+5x^3) + \right. \\ \left(5 \times 3^{3/4} (-1+x) x^4 \sqrt{1+x+x^2} \left(-i\sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}} \right], (-1)^{1/3} \right] + \right. \right. \\ \left. \left. (-1)^{1/3} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) \right) / \left(\sqrt{(-1)^{5/6}(-1+x)} \right)$$

Problem 478: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{-1+x^3}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\frac{2}{3} \text{ArcTan}[\sqrt{-1+x^3}]$$

Result (type 3, 36 leaves):

$$\frac{2 \sqrt{-1+x^3} \text{ArcTanh}[\sqrt{1-x^3}]}{3 \sqrt{1-x^3}}$$

Problem 482: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 153 leaves, 3 steps):

$$\frac{16}{55} x \sqrt{-1+x^3} + \frac{2}{11} x^4 \sqrt{-1+x^3} - \left(32 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \left(55 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)$$

Result (type 4, 91 leaves):

$$\frac{1}{165 \sqrt{-1+x^3}} 2 \left(3 x (-8 + 3 x^3 + 5 x^6) + 16 i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 483: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 137 leaves, 2 steps):

$$\frac{2}{5} x \sqrt{-1+x^3} - \left(4 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \left(5 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)$$

Result (type 4, 84 leaves):

$$\frac{1}{15 \sqrt{-1+x^3}} 2 \left(3 x (-1+x^3) + 2 i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 484: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 120 leaves, 1 step):

$$- \left(\left(2 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \left(3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) \right)$$

Result (type 4, 71 leaves):

$$\frac{1}{3^{1/4} \sqrt{-1+x^3}} 2 i \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 485: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{-1+x^3}} dx$$

Optimal (type 4, 139 leaves, 2 steps):

$$\frac{\sqrt{-1+x^3}}{2x^2} - \left(\sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) /$$

$$\left(2 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)$$

Result (type 4, 90 leaves):

$$\frac{\sqrt{-1+x^3}}{2x^2} + \frac{1}{2 \times 3^{1/4} \sqrt{-1+x^3}}$$

$$i \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 486: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{-1+x^3}} dx$$

Optimal (type 4, 155 leaves, 3 steps):

$$\frac{\sqrt{-1+x^3}}{5x^5} + \frac{7\sqrt{-1+x^3}}{20x^2} -$$

$$\left(7\sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) /$$

$$\left(20 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)$$

Result (type 4, 93 leaves):

$$\frac{1}{60x^5 \sqrt{-1+x^3}} \left(-12 - 9x^3 + 21x^6 +$$

$$7i 3^{3/4} \sqrt{(-1)^{5/6} (-1+x)} x^5 \sqrt{1+x+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 487: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{80\sqrt{-1+x^3}}{91(1-\sqrt{3}-x)} + \frac{20}{91}x^2\sqrt{-1+x^3} + \frac{2}{13}x^5\sqrt{-1+x^3} + \\
 & \left(40 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(91 \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) - \\
 & \frac{80\sqrt{2}(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{91 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
 \end{aligned}$$

Result (type 4, 142 leaves):

$$\begin{aligned}
 & \frac{1}{273\sqrt{-1+x^3}} 2 \left(3x^2(-1+x^3)(10+7x^3) + 40(-1)^{1/6}3^{3/4}\sqrt{(-1)^{5/6}(-1+x)} \right. \\
 & \left. \sqrt{1+x+x^2} \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\
 & \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 488: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{8\sqrt{-1+x^3}}{7(1-\sqrt{3}-x)} + \frac{2}{7}x^2\sqrt{-1+x^3} + \\
 & \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(7 \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) - \\
 & \frac{8\sqrt{2}(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{7 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
 \end{aligned}$$

Result (type 4, 135 leaves):

$$\begin{aligned}
 & \frac{1}{21\sqrt{-1+x^3}} 2 \left(3x^2(-1+x^3) + 4(-1)^{1/6} 3^{3/4} \sqrt{(-1)^{5/6}(-1+x)} \right. \\
 & \left. \sqrt{1+x+x^2} \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\
 & \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 489: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{-1+x^3}} dx$$

Optimal (type 4, 255 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \\
 & \left(3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) - \\
 & \frac{2\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
 \end{aligned}$$

Result (type 4, 120 leaves):

$$\begin{aligned}
 & \frac{1}{3^{1/4} \sqrt{-1+x^3}} 2 (-1)^{1/6} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \\
 & \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
 & \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)
 \end{aligned}$$

Problem 490: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{-1+x^3}} dx$$

Optimal (type 4, 269 leaves, 4 steps):

$$\frac{\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt{-1+x^3}}{x} - \left(3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \left(2 \sqrt{\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) + \frac{\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 130 leaves):

$$\frac{\sqrt{-1+x^3}}{x} + \frac{1}{3^{1/4} \sqrt{-1+x^3}} - (-1)^{2/3} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 491: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{-1+x^3}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{5 \sqrt{-1+x^3}}{8(1-\sqrt{3}-x)} + \frac{\sqrt{-1+x^3}}{4x^4} + \frac{5 \sqrt{-1+x^3}}{8x} -$$

$$\left(5 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) /$$

$$\left(16 \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) +$$

$$\frac{5(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{4\sqrt{2} 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 140 leaves):

$$\frac{1}{24 \sqrt{-1+x^3}} \left(\frac{3(-1+x^3)(2+5x^3)}{x^4} + \right.$$

$$\left. \left(5 \times 3^{3/4} (-1+x) \sqrt{1+x+x^2} \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \right.$$

$$\left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-ix}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right) / \left(\sqrt{(-1)^{5/6}(-1+x)} \right)$$

Problem 496: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{-1-x^3}} dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\frac{2}{3} \text{ArcTan}\left[\sqrt{-1-x^3}\right]$$

Result (type 3, 34 leaves):

$$\frac{2 \sqrt{-1-x^3} \text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3 \sqrt{1+x^3}}$$

Problem 500: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 149 leaves, 3 steps):

$$\frac{16}{55} x \sqrt{-1-x^3} - \frac{2}{11} x^4 \sqrt{-1-x^3} + \left(32 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) / \left(55 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 115 leaves):

$$\frac{1}{165 \sqrt{-1-x^3}} 2 \left(3 x (-8-3 x^3+5 x^6) + 16 (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6}+i x} \sqrt{1-(-1)^{2/3} x - (-1)^{1/3} x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 501: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 131 leaves, 2 steps):

$$-\frac{2}{5} x \sqrt{-1-x^3} - \left(4 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) / \left(5 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 107 leaves):

$$\frac{1}{15 \sqrt{-1-x^3}} \left(6 (x+x^4) - 4 (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + i x} \right. \\ \left. \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 112 leaves, 1 step):

$$\left(2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right] \right) / \\ \left(3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 95 leaves):

$$\frac{1}{3^{1/4} \sqrt{-1-x^3}} 2 (-1)^{5/6} \sqrt{-(-1)^{5/6} + i x} \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \\ \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)}}{3^{1/4}} \right], (-1)^{1/3} \right]$$

Problem 503: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{-1-x^3}} dx$$

Optimal (type 4, 133 leaves, 2 steps):

$$\frac{\sqrt{-1-x^3}}{2x^2} - \left(\sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right] \right) / \\ \left(2 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 111 leaves):

$$-\frac{1}{6x^2\sqrt{-1-x^3}} \left(3 + 3x^3 + (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + ix} x^2 \right. \\ \left. \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3} + x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 504: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{-1-x^3}} dx$$

Optimal (type 4, 151 leaves, 3 steps):

$$\frac{\sqrt{-1-x^3}}{5x^5} - \frac{7\sqrt{-1-x^3}}{20x^2} + \\ \left(7\sqrt{2-\sqrt{3}}(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right] \right) / \\ \left(20 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 117 leaves):

$$\frac{1}{60x^5\sqrt{-1-x^3}} \left(-12 + 9x^3 + 21x^6 + 7(-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + ix} x^5 \right. \\ \left. \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3} + x)}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 505: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$\frac{20}{91} x^2 \sqrt{-1-x^3} - \frac{2}{13} x^5 \sqrt{-1-x^3} - \frac{80 \sqrt{-1-x^3}}{91 (1-\sqrt{3}+x)} +$$

$$\left(40 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) /$$

$$\left(91 \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) -$$

$$\frac{80 \sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{91 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 164 leaves):

$$\frac{1}{273 \sqrt{-1-x^3}}$$

$$2 \left(3 x^2 (1+x^3) (-10+7 x^3) + 40 (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + i x} \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \right.$$

$$\left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 506: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 264 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2}{7} x^2 \sqrt{-1-x^3} + \frac{8 \sqrt{-1-x^3}}{7 (1-\sqrt{3}+x)} - \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) / \\
& \left(7 \sqrt{\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) + \\
& \frac{8 \sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{7 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}
\end{aligned}$$

Result (type 4, 157 leaves):

$$\begin{aligned}
& \frac{1}{21 \sqrt{-1-x^3}} 2 \left(3 x^2 (1+x^3) - 4 (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + i x} \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \right. \\
& \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
\end{aligned}$$

Problem 507: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{-1-x^3}} dx$$

Optimal (type 4, 239 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \\
 & \left(3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(\sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) - \\
 & \frac{2\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}
 \end{aligned}$$

Result (type 4, 142 leaves):

$$\begin{aligned}
 & \frac{1}{3^{1/4} \sqrt{-1-x^3}} 2 (-1)^{5/6} \sqrt{-(-1)^{5/6} + i x} \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \\
 & \left(-i \sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
 & \left. (-1)^{1/3} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)
 \end{aligned}$$

Problem 508: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{-1-x^3}} dx$$

Optimal (type 4, 257 leaves, 4 steps):

$$\frac{\sqrt{-1-x^3}}{x} - \frac{\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \left(3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) / \left(2 \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) - \frac{\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 156 leaves):

$$\frac{1}{3\sqrt{-1-x^3}} \left(-\frac{3(1+x^3)}{x} + (-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + i x} \sqrt{1 - (-1)^{2/3} x - (-1)^{1/3} x^2} \right. \\ \left. \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\ \left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 509: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{-1-x^3}} dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$\frac{\sqrt{-1-x^3}}{4x^4} - \frac{5\sqrt{-1-x^3}}{8x} + \frac{5\sqrt{-1-x^3}}{8(1-\sqrt{3}+x)} -$$

$$\left(5 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right] \right) /$$

$$\left(16 \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) +$$

$$\frac{5(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]}{4\sqrt{2} 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 164 leaves):

$$\frac{1}{24\sqrt{-1-x^3}} \left(\frac{3(1+x^3)(-2+5x^3)}{x^4} - 5(-1)^{5/6} 3^{3/4} \sqrt{-(-1)^{5/6} + ix} \sqrt{1 - (-1)^{2/3}x - (-1)^{1/3}x^2} \right.$$

$$\left. \left(-i\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left. \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}((-1)^{2/3}+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 514: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{1/3}}{x} dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$(a+bx^3)^{1/3} - \frac{a^{1/3} \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx^3)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}} - \frac{1}{2}a^{1/3} \text{Log}[x] + \frac{1}{2}a^{1/3} \text{Log}[a^{1/3} - (a+bx^3)^{1/3}]$$

Result (type 5, 61 leaves):

$$\frac{2 (a + b x^3) - a \left(1 + \frac{a}{b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^3}\right]}{2 (a + b x^3)^{2/3}}$$

Problem 515: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^4} dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{(a + b x^3)^{1/3}}{3 x^3} - \frac{b \text{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{2/3}} - \frac{b \text{Log}[x]}{6 a^{2/3}} + \frac{b \text{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]}{6 a^{2/3}}$$

Result (type 5, 67 leaves):

$$\frac{-2 (a + b x^3) - b \left(1 + \frac{a}{b x^3}\right)^{2/3} x^3 \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^3}\right]}{6 x^3 (a + b x^3)^{2/3}}$$

Problem 516: Result unnecessarily involves higher level functions.

$$\int x^4 (a + b x^3)^{1/3} dx$$

Optimal (type 3, 173 leaves, 9 steps):

$$\frac{a x^2 (a + b x^3)^{1/3}}{18 b} + \frac{1}{6} x^5 (a + b x^3)^{1/3} + \frac{a^2 \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{5/3}} +$$

$$\frac{a^2 \text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{27 b^{5/3}} - \frac{a^2 \text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{54 b^{5/3}}$$

Result (type 5, 78 leaves):

$$\frac{1}{18 b (a + b x^3)^{2/3}} x^2 \left(a^2 + 4 a b x^3 + 3 b^2 x^6 - a^2 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right] \right)$$

Problem 517: Result unnecessarily involves higher level functions.

$$\int x (a + b x^3)^{1/3} dx$$

Optimal (type 3, 145 leaves, 8 steps):

$$\frac{1}{3} x^2 (a + b x^3)^{1/3} - \frac{a \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} - \frac{a \text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{2/3}} + \frac{a \text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 b^{2/3}}$$

Result (type 5, 63 leaves):

$$\frac{x^2 \left(2 (a + b x^3) + a \left(1 + \frac{b x^3}{a} \right)^{2/3} \text{Hypergeometric2F1} \left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a} \right] \right)}{6 (a + b x^3)^{2/3}}$$

Problem 518: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^2} dx$$

Optimal (type 3, 138 leaves, 8 steps):

$$-\frac{(a + b x^3)^{1/3}}{x} - \frac{b^{1/3} \text{ArcTan} \left[\frac{1 + \frac{2b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3}} - \frac{1}{3} b^{1/3} \text{Log} \left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}} \right] + \frac{1}{6} b^{1/3} \text{Log} \left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}} \right]$$

Result (type 5, 66 leaves):

$$\frac{-2 (a + b x^3) + b x^3 \left(1 + \frac{b x^3}{a} \right)^{2/3} \text{Hypergeometric2F1} \left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a} \right]}{2 x (a + b x^3)^{2/3}}$$

Problem 523: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x^3)^{1/3} dx$$

Optimal (type 5, 33 leaves, 2 steps):

$$\frac{x (a + b x^3)^{4/3} \text{Hypergeometric2F1} \left[1, \frac{5}{3}, \frac{4}{3}, -\frac{b x^3}{a} \right]}{a}$$

Result (type 6, 196 leaves):

$$\left(3 \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right) (a + b x^3)^{1/3} \text{AppellF1} \left[\frac{4}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{7}{3}, -\frac{i \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)}{\sqrt{3} a^{1/3}}, \frac{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}}{3i + \sqrt{3}} \right] \right) / \left(4 \times 2^{1/3} b^{1/3} \left(\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \right)^{1/3} \left(\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}} \right)}{3i + \sqrt{3}} \right)^{1/3} \right)$$

Problem 525: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^{1/3}}{x^6} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{(a + b x^3)^{4/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 1, -\frac{2}{3}, -\frac{b x^3}{a}\right]}{5 a x^5}$$

Result (type 5, 83 leaves):

$$\frac{\left(-2 a^2 - 3 a b x^3 - b^2 x^6 - b^2 x^6 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]\right)}{\left(10 a x^5 (a + b x^3)^{2/3}\right)}$$

Problem 530: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x} dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$\frac{1}{2} (a + b x^3)^{2/3} + \frac{a^{2/3} \text{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3}} - \frac{1}{2} a^{2/3} \text{Log}[x] + \frac{1}{2} a^{2/3} \text{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]$$

Result (type 5, 58 leaves):

$$\frac{a + b x^3 - 2 a \left(1 + \frac{a}{b x^3}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x^3}\right]}{2 (a + b x^3)^{1/3}}$$

Problem 531: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^4} dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{(a + b x^3)^{2/3}}{3 x^3} + \frac{2 b \text{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{1/3}} - \frac{b \text{Log}[x]}{3 a^{1/3}} + \frac{b \text{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]}{3 a^{1/3}}$$

Result (type 5, 67 leaves):

$$\frac{-a - b x^3 - 2 b \left(1 + \frac{a}{b x^3}\right)^{1/3} x^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x^3}\right]}{3 x^3 (a + b x^3)^{1/3}}$$

Problem 532: Result more than twice size of optimal antiderivative.

$$\int x^4 (a + b x^3)^{2/3} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{x^5 (a + b x^3)^{5/3} \text{Hypergeometric2F1}\left[1, \frac{10}{3}, \frac{8}{3}, -\frac{b x^3}{a}\right]}{5 a}$$

Result (type 5, 78 leaves):

$$\frac{1}{14 b (a + b x^3)^{1/3}} x^2 \left(a^2 + 3 a b x^3 + 2 b^2 x^6 - a^2 \left(1 + \frac{b x^3}{a} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right] \right)$$

Problem 535: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^{2/3}}{x^5} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{(a + b x^3)^{5/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, -\frac{1}{3}, -\frac{b x^3}{a}\right]}{4 a x^4}$$

Result (type 5, 82 leaves):

$$\frac{\left(-a^2 - 3 a b x^3 - 2 b^2 x^6 + b^2 x^6 \left(1 + \frac{b x^3}{a} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right] \right)}{\left(4 a x^4 (a + b x^3)^{1/3} \right)}$$

Problem 537: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x^3)^{2/3} dx$$

Optimal (type 3, 91 leaves, 2 steps):

$$\frac{1}{3} x (a + b x^3)^{2/3} + \frac{2 a \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{1/3}} - \frac{a \text{Log}\left[-b^{1/3} x + (a + b x^3)^{1/3}\right]}{3 b^{1/3}}$$

Result (type 6, 196 leaves):

$$\left(3 \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right) (a + b x^3)^{2/3} \right. \\ \left. \text{AppellF1} \left[\frac{5}{3}, -\frac{2}{3}, -\frac{2}{3}, \frac{8}{3}, -\frac{i \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)}{\sqrt{3} a^{1/3}}, \frac{i + \sqrt{3} - \frac{2 i b^{1/3} x}{a^{1/3}}}{3 i + \sqrt{3}} \right] \right) / \\ \left(5 \times 2^{2/3} b^{1/3} \left(\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \right)^{2/3} \left(\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}} \right)}{3 i + \sqrt{3}} \right)^{2/3} \right)$$

Problem 547: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x^3)^{1/3}} dx$$

Optimal (type 3, 83 leaves, 5 steps):

$$\frac{\text{ArcTan} \left[\frac{a^{1/3} + 2 (a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{1/3}} - \frac{\text{Log}[x]}{2 a^{1/3}} + \frac{\text{Log} \left[a^{1/3} - (a + b x^3)^{1/3} \right]}{2 a^{1/3}}$$

Result (type 5, 46 leaves):

$$\frac{\left(1 + \frac{a}{b x^3} \right)^{1/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x^3} \right]}{(a + b x^3)^{1/3}}$$

Problem 548: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a + b x^3)^{1/3}} dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{(a + b x^3)^{2/3}}{3 a x^3} - \frac{b \text{ArcTan} \left[\frac{a^{1/3} + 2 (a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}} \right]}{3 \sqrt{3} a^{4/3}} + \frac{b \text{Log}[x]}{6 a^{4/3}} - \frac{b \text{Log} \left[a^{1/3} - (a + b x^3)^{1/3} \right]}{6 a^{4/3}}$$

Result (type 5, 69 leaves):

$$\frac{-a - b x^3 + b \left(1 + \frac{a}{b x^3} \right)^{1/3} x^3 \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x^3} \right]}{3 a x^3 (a + b x^3)^{1/3}}$$

Problem 549: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{(a + b x^3)^{1/3}} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{x^8 (a + b x^3)^{2/3} \text{Hypergeometric2F1}\left[1, \frac{10}{3}, \frac{11}{3}, -\frac{b x^3}{a}\right]}{8 a}$$

Result (type 5, 80 leaves):

$$\frac{1}{28 b^2 (a + b x^3)^{1/3}} x^2 \left(-5 a^2 - a b x^3 + 4 b^2 x^6 + 5 a^2 \left(1 + \frac{b x^3}{a} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right] \right)$$

Problem 553: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 (a + b x^3)^{1/3}} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-\frac{(a + b x^3)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3}, 1, -\frac{1}{3}, -\frac{b x^3}{a}\right]}{4 a x^4}$$

Result (type 5, 82 leaves):

$$\frac{\left(-a^2 + a b x^3 + 2 b^2 x^6 - b^2 x^6 \left(1 + \frac{b x^3}{a} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right] \right)}{\left(4 a^2 x^4 (a + b x^3)^{1/3} \right)}$$

Problem 564: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x^3)^{2/3}} dx$$

Optimal (type 3, 84 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3}} - \frac{\text{Log}[x]}{2 a^{2/3}} + \frac{\text{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]}{2 a^{2/3}}$$

Result (type 5, 48 leaves):

$$-\frac{\left(1 + \frac{a}{b x^3} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^3}\right]}{2 (a + b x^3)^{2/3}}$$

Problem 565: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a + b x^3)^{2/3}} dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{(a + b x^3)^{1/3}}{3 a x^3} + \frac{2 b \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x^3)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3}} + \frac{b \operatorname{Log}[x]}{3 a^{5/3}} - \frac{b \operatorname{Log}\left[a^{1/3} - (a + b x^3)^{1/3}\right]}{3 a^{5/3}}$$

Result (type 5, 69 leaves):

$$\frac{-a - b x^3 + b \left(1 + \frac{a}{b x^3}\right)^{2/3} x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^3}\right]}{3 a x^3 (a + b x^3)^{2/3}}$$

Problem 566: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(a + b x^3)^{2/3}} dx$$

Optimal (type 3, 148 leaves, 8 steps):

$$\frac{x^2 (a + b x^3)^{1/3}}{3 b} + \frac{2 a \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}} + \frac{2 a \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{5/3}} - \frac{a \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{5/3}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(a + b x^3 - a \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]\right)}{3 b (a + b x^3)^{2/3}}$$

Problem 567: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a + b x^3)^{2/3}} dx$$

Optimal (type 3, 72 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{\operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 52 leaves):

$$\frac{x^2 \left(\frac{a + b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}}$$

Problem 572: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6}{(a + b x^3)^{2/3}} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{x^7 (a + b x^3)^{1/3} \text{Hypergeometric2F1}\left[1, \frac{8}{3}, \frac{10}{3}, -\frac{b x^3}{a}\right]}{7 a}$$

Result (type 5, 78 leaves):

$$\frac{1}{5 b^2 (a + b x^3)^{2/3}} \left(-2 a^2 x - a b x^4 + b^2 x^7 + 2 a^2 x \left(1 + \frac{b x^3}{a} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right] \right)$$

Problem 574: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{2/3}} dx$$

Optimal (type 5, 33 leaves, 2 steps):

$$\frac{x (a + b x^3)^{1/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}\right]}{a}$$

Result (type 5, 177 leaves):

$$\frac{1}{b^{1/3} (a + b x^3)^{2/3}} 3 \times 2^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right) \left(\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \right)^{2/3} \\ \left(\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}} \right)}{3 i + \sqrt{3}} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{(1 + i \sqrt{3}) a^{1/3} + (1 - i \sqrt{3}) b^{1/3} x}{2 (a^{1/3} + b^{1/3} x)}\right]$$

Problem 576: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 (a + b x^3)^{2/3}} dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{(a + b x^3)^{1/3} \text{Hypergeometric2F1}\left[-\frac{4}{3}, 1, -\frac{2}{3}, -\frac{b x^3}{a}\right]}{5 a x^5}$$

Result (type 5, 82 leaves):

$$\frac{\left(-a^2 + a b x^3 + 2 b^2 x^6 + 2 b^2 x^6 \left(1 + \frac{b x^3}{a} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right] \right)}{\left(5 a^2 x^5 (a + b x^3)^{2/3} \right)}$$

Problem 579: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(1 - x^3)^{2/3}} dx$$

Optimal (type 3, 49 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{2} \text{Log}\left[x + (1-x^3)^{1/3}\right]$$

Result (type 5, 20 leaves):

$$\frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right]$$

Problem 633: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + b x^4)^3 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^4)^4}{16 b}$$

Result (type 1, 43 leaves):

$$\frac{a^3 x^4}{4} + \frac{3}{8} a^2 b x^8 + \frac{1}{4} a b^2 x^{12} + \frac{b^3 x^{16}}{16}$$

Problem 774: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{a + c x^4} dx$$

Optimal (type 4, 127 leaves, 3 steps):

$$\frac{2 a x \sqrt{a + c x^4}}{21 c} + \frac{1}{7} x^5 \sqrt{a + c x^4} - \frac{a^{7/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{21 c^{5/4} \sqrt{a + c x^4}}$$

Result (type 4, 106 leaves):

$$\frac{2 a^2 x + 5 a c x^5 + 3 c^2 x^9 + \frac{2 i a^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{21 c \sqrt{a + c x^4}}$$

Problem 775: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + c x^4} dx$$

Optimal (type 4, 105 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{a + c x^4} + \frac{a^{3/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 89 leaves):

$$x (a + c x^4) - \frac{2 i a \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$3 \sqrt{a + c x^4}$$

Problem 776: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{x^4} dx$$

Optimal (type 4, 107 leaves, 2 steps):

$$-\frac{\sqrt{a + c x^4}}{3 x^3} + \frac{c^{3/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 a^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 92 leaves):

$$-\frac{a + c x^4}{x^3} - \frac{2 i c \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$3 \sqrt{a + c x^4}$$

Problem 777: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{x^8} dx$$

Optimal (type 4, 129 leaves, 3 steps):

$$-\frac{\sqrt{a + c x^4}}{7 x^7} - \frac{2 c \sqrt{a + c x^4}}{21 a x^3} - \frac{c^{7/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{21 a^{5/4} \sqrt{a + c x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{3a^2}{x^7} - \frac{5ac}{x^3} - 2c^2x + \frac{2ic^2\sqrt{1+\frac{cx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right]x, -1\right]}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{21a\sqrt{a+cx^4}}$$

Problem 778: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a+cx^4} \, dx$$

Optimal (type 4, 234 leaves, 4 steps):

$$\frac{1}{5}x^3\sqrt{a+cx^4} + \frac{2ax\sqrt{a+cx^4}}{5\sqrt{c}(\sqrt{a}+\sqrt{c}x^2)} - \frac{2a^{5/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{5c^{3/4}\sqrt{a+cx^4}} + \frac{a^{5/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{5c^{3/4}\sqrt{a+cx^4}}$$

Result (type 4, 121 leaves):

$$\frac{1}{5\sqrt{a+cx^4}} \left(x^3(a+cx^4) + \frac{1}{\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}} 2ia\sqrt{1+\frac{cx^4}{a}} \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right]x, -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right]x, -1\right] \right) \right)$$

Problem 779: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+cx^4}}{x^2} \, dx$$

Optimal (type 4, 224 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+c x^4}}{x} + \frac{2\sqrt{c} x \sqrt{a+c x^4}}{\sqrt{a} + \sqrt{c} x^2} - \frac{1}{\sqrt{a+c x^4}} \\
 & 2 a^{1/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \\
 & \frac{1}{\sqrt{a+c x^4}} a^{1/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]
 \end{aligned}$$

Result (type 4, 119 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{a+c x^4}} \left(-\frac{a+c x^4}{x} + \frac{1}{\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}} 2 i c \sqrt{1 + \frac{c x^4}{a}} \right. \\
 & \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right)
 \end{aligned}$$

Problem 780: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{x^6} dx$$

Optimal (type 4, 258 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+c x^4}}{5 x^5} - \frac{2 c \sqrt{a+c x^4}}{5 a x} + \frac{2 c^{3/2} x \sqrt{a+c x^4}}{5 a (\sqrt{a} + \sqrt{c} x^2)} - \\
 & \frac{2 c^{5/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{3/4} \sqrt{a+c x^4}} + \\
 & \frac{c^{5/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{3/4} \sqrt{a+c x^4}}
 \end{aligned}$$

Result (type 4, 133 leaves):

$$\begin{aligned}
 & \frac{1}{5 \sqrt{a+c x^4}} \left(-\frac{(a+c x^4)(a+2 c x^4)}{a x^5} - 2 i \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} c \sqrt{1 + \frac{c x^4}{a}} \right. \\
 & \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right)
 \end{aligned}$$

Problem 794: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (a + c x^4)^{3/2} dx$$

Optimal (type 4, 148 leaves, 4 steps):

$$\frac{4 a^2 x \sqrt{a + c x^4}}{77 c} + \frac{6}{77} a x^5 \sqrt{a + c x^4} + \frac{1}{11} x^5 (a + c x^4)^{3/2} - \frac{2 a^{11/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{77 c^{5/4} \sqrt{a + c x^4}}$$

Result (type 4, 117 leaves):

$$\frac{1}{77 c \sqrt{a + c x^4}} \left(4 a^3 x + 17 a^2 c x^5 + 20 a c^2 x^9 + 7 c^3 x^{13} + \frac{4 i a^3 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} \right)$$

Problem 795: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + c x^4)^{3/2} dx$$

Optimal (type 4, 122 leaves, 3 steps):

$$\frac{2}{7} a x \sqrt{a + c x^4} + \frac{1}{7} x (a + c x^4)^{3/2} + \frac{2 a^{7/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{7 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 102 leaves):

$$\frac{3 a^2 x + 4 a c x^5 + c^2 x^9 - \frac{4 i a^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{7 \sqrt{a + c x^4}}$$

Problem 796: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + c x^4)^{3/2}}{x^4} dx$$

Optimal (type 4, 124 leaves, 3 steps):

$$\frac{2}{3} c x \sqrt{a+c x^4} - \frac{(a+c x^4)^{3/2}}{3 x^3} + \frac{1}{3 \sqrt{a+c x^4}}$$

$$2 a^{3/4} c^{3/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 96 leaves):

$$-\frac{a^2}{x^3} + c^2 x^5 - \frac{4 i a c \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$3 \sqrt{a+c x^4}$$

Problem 797: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+c x^4)^{3/2}}{x^8} dx$$

Optimal (type 4, 126 leaves, 3 steps):

$$-\frac{2 c \sqrt{a+c x^4}}{7 x^3} - \frac{(a+c x^4)^{3/2}}{7 x^7} + \frac{2 c^{7/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{7 a^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 106 leaves):

$$-\frac{a^2+4 a c x^4+3 c^2 x^8}{x^7} - \frac{4 i c^2 \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$7 \sqrt{a+c x^4}$$

Problem 798: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (a+c x^4)^{3/2} dx$$

Optimal (type 4, 255 leaves, 5 steps):

$$\frac{2}{15} a x^3 \sqrt{a+c x^4} + \frac{4 a^2 x \sqrt{a+c x^4}}{15 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{9} x^3 (a+c x^4)^{3/2} -$$

$$\frac{4 a^{9/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}} +$$

$$\frac{2 a^{9/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 133 leaves):

$$\frac{1}{45 \sqrt{a+c x^4}} \left((a+c x^4) (11 a x^3 + 5 c x^7) + \frac{1}{\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2}} 12 i a^2 \sqrt{1 + \frac{c x^4}{a}} \right.$$

$$\left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right)$$

Problem 799: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+c x^4)^{3/2}}{x^2} dx$$

Optimal (type 4, 251 leaves, 5 steps):

$$\frac{6}{5} c x^3 \sqrt{a+c x^4} + \frac{12 a \sqrt{c} x \sqrt{a+c x^4}}{5 (\sqrt{a} + \sqrt{c} x^2)} - \frac{(a+c x^4)^{3/2}}{x} - \frac{1}{5 \sqrt{a+c x^4}}$$

$$12 a^{5/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{1}{5 \sqrt{a+c x^4}} 6 a^{5/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 136 leaves):

$$\left(-\frac{a}{x} + \frac{c x^3}{5}\right) \sqrt{a + c x^4} + \left(12 i a c \sqrt{1 + \frac{c x^4}{a}}\right. \\ \left.\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]\right)\right) / \left(5\right. \\ \left.\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{a + c x^4}\right)$$

Problem 800: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + c x^4)^{3/2}}{x^6} dx$$

Optimal (type 4, 252 leaves, 5 steps):

$$-\frac{6 c \sqrt{a + c x^4}}{5 x} + \frac{12 c^{3/2} x \sqrt{a + c x^4}}{5 (\sqrt{a} + \sqrt{c} x^2)} - \frac{(a + c x^4)^{3/2}}{5 x^5} - \frac{1}{5 \sqrt{a + c x^4}} \\ 12 a^{1/4} c^{5/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \\ \frac{1}{5 \sqrt{a + c x^4}} 6 a^{1/4} c^{5/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 132 leaves):

$$\frac{1}{5 \sqrt{a + c x^4}} \left(-\frac{(a + c x^4)(a + 7 c x^4)}{x^5} + \frac{1}{\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2}} 12 i c^2 \sqrt{1 + \frac{c x^4}{a}}\right. \\ \left.\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]\right)\right)$$

Problem 801: Result unnecessarily involves imaginary or complex numbers.

$$\int (1 + x^4)^{3/2} dx$$

Optimal (type 4, 72 leaves, 3 steps):

$$\frac{2}{7} x \sqrt{1 + x^4} + \frac{1}{7} x (1 + x^4)^{3/2} + \frac{2 (1 + x^2) \sqrt{\frac{1 + x^4}{(1 + x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{7 \sqrt{1 + x^4}}$$

Result (type 4, 55 leaves):

$$\frac{1}{7 \sqrt{1+x^4}} \left(3 x + 4 x^5 + x^9 - 4 (-1)^{1/4} \sqrt{1+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 807: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{1+x^4} \, dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{1+x^4} + \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{3 \sqrt{1+x^4}}$$

Result (type 4, 48 leaves):

$$\frac{x + x^5 - 2 (-1)^{1/4} \sqrt{1+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right]}{3 \sqrt{1+x^4}}$$

Problem 819: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{\sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 130 leaves, 3 steps):

$$-\frac{5 a x \sqrt{a+b x^4}}{21 b^2} + \frac{x^5 \sqrt{a+b x^4}}{7 b} + \frac{5 a^{7/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{42 b^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-5 a^2 x - 2 a b x^5 + 3 b^2 x^9 - \frac{5 i a^2 \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{21 b^2 \sqrt{a+b x^4}}$$

Problem 820: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{x \sqrt{a + b x^4}}{3 b} - \frac{a^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 b^{5/4} \sqrt{a + b x^4}}$$

Result (type 4, 92 leaves):

$$\frac{x (a + b x^4) + \frac{i a \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{3 b \sqrt{a + b x^4}}$$

Problem 821: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{(\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} \sqrt{a + b x^4}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a + b x^4}}$$

Problem 822: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a + b x^4}} dx$$

Optimal (type 4, 110 leaves, 2 steps):

$$\frac{\frac{\sqrt{a + b x^4}}{3 a x^3} - \frac{b^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 a^{5/4} \sqrt{a + b x^4}}}{3 a x^3}$$

Result (type 4, 95 leaves):

$$\frac{-\frac{a + b x^4}{x^3} + \frac{i b \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{3 a \sqrt{a + b x^4}}$$

Problem 823: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 \sqrt{a + b x^4}} dx$$

Optimal (type 4, 132 leaves, 3 steps):

$$-\frac{\sqrt{a + b x^4}}{7 a x^7} + \frac{5 b \sqrt{a + b x^4}}{21 a^2 x^3} + \frac{5 b^{7/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{42 a^{9/4} \sqrt{a + b x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{3 a^2}{x^7} + \frac{2 a b}{x^3} + 5 b^2 x - \frac{5 i b^2 \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{21 a^2 \sqrt{a + b x^4}}$$

Problem 824: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{\sqrt{a + b x^4}} dx$$

Optimal (type 4, 261 leaves, 5 steps):

$$-\frac{7 a x^3 \sqrt{a + b x^4}}{45 b^2} + \frac{x^7 \sqrt{a + b x^4}}{9 b} + \frac{7 a^2 x \sqrt{a + b x^4}}{15 b^{5/2} (\sqrt{a} + \sqrt{b} x^2)} - \frac{7 a^{9/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{11/4} \sqrt{a + b x^4}} + \frac{7 a^{9/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{30 b^{11/4} \sqrt{a + b x^4}}$$

Result (type 4, 136 leaves):

$$\frac{1}{45 b^2 \sqrt{a + b x^4}} \left((a + b x^4) (-7 a x^3 + 5 b x^7) + \frac{1}{\left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2}} 21 i a^2 \sqrt{1 + \frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) \right)$$

Problem 825: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a + b x^4}} dx$$

Optimal (type 4, 237 leaves, 4 steps):

$$\frac{x^3 \sqrt{a + b x^4}}{5 b} - \frac{3 a x \sqrt{a + b x^4}}{5 b^{3/2} (\sqrt{a} + \sqrt{b} x^2)} +$$

$$\frac{3 a^{5/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{a + b x^4}} -$$

$$\frac{3 a^{5/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{10 b^{7/4} \sqrt{a + b x^4}}$$

Result (type 4, 168 leaves):

$$\frac{x^3 \sqrt{a + b x^4}}{5 b} - \left(3 a^{3/2} \sqrt{1 - \frac{i \sqrt{b} x^2}{\sqrt{a}}} \sqrt{1 + \frac{i \sqrt{b} x^2}{\sqrt{a}}} \right.$$

$$\left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) \right) /$$

$$\left(5 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^{3/2} \sqrt{a + b x^4} \right)$$

Problem 826: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + b x^4}} dx$$

Optimal (type 4, 210 leaves, 3 steps):

$$\frac{x \sqrt{a + b x^4}}{\sqrt{b} (\sqrt{a} + \sqrt{b} x^2)} - \frac{a^{1/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} \sqrt{a + b x^4}} +$$

$$\frac{a^{1/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 b^{3/4} \sqrt{a + b x^4}}$$

Result (type 4, 104 leaves):

$$\left(i \sqrt{1 + \frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x \right], -1 \right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x \right], -1 \right] \right) \right) / \left(\left(\frac{i \sqrt{b}}{\sqrt{a}} \right)^{3/2} \sqrt{a + b x^4} \right)$$

Problem 827: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + b x^4}} dx$$

Optimal (type 4, 232 leaves, 4 steps):

$$-\frac{\sqrt{a + b x^4}}{a x} + \frac{\sqrt{b} x \sqrt{a + b x^4}}{a (\sqrt{a} + \sqrt{b} x^2)} - \frac{b^{1/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{a^{3/4} \sqrt{a + b x^4}} + \frac{b^{1/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{3/4} \sqrt{a + b x^4}}$$

Result (type 4, 121 leaves):

$$\frac{1}{\sqrt{a + b x^4}} \left(-\frac{a + b x^4}{a x} - i \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{1 + \frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x \right], -1 \right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x \right], -1 \right] \right) \right)$$

Problem 828: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{a + b x^4}} dx$$

Optimal (type 4, 261 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^4}}{5ax^5} + \frac{3b\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}x\sqrt{a+bx^4}}{5a^2(\sqrt{a}+\sqrt{b}x^2)} + \\
 & \frac{3b^{5/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{5a^{7/4}\sqrt{a+bx^4}} - \\
 & \frac{3b^{5/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{10a^{7/4}\sqrt{a+bx^4}}
 \end{aligned}$$

Result (type 4, 135 leaves):

$$\begin{aligned}
 & \frac{1}{5a^2\sqrt{a+bx^4}} \left(\frac{(a+bx^4)(-a+3bx^4)}{x^5} + 3i a \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} b \sqrt{1+\frac{bx^4}{a}} \right. \\
 & \left. \left(\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] - \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] \right) \right)
 \end{aligned}$$

Problem 839: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{\sqrt{a-bx^4}} dx$$

Optimal (type 4, 100 leaves, 4 steps):

$$-\frac{5ax\sqrt{a-bx^4}}{21b^2} - \frac{x^5\sqrt{a-bx^4}}{7b} + \frac{5a^{9/4}\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}x}{a^{1/4}}\right], -1\right]}{21b^{9/4}\sqrt{a-bx^4}}$$

Result (type 4, 122 leaves):

$$\begin{aligned}
 & \left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x(-5a^2+2abx^4+3b^2x^8) - 5ia^2\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right], -1\right] \right) / \\
 & \left(21\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}b^2\sqrt{a-bx^4} \right)
 \end{aligned}$$

Problem 840: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a-bx^4}} dx$$

Optimal (type 4, 77 leaves, 3 steps):

$$-\frac{x \sqrt{a - b x^4}}{3 b} + \frac{a^{5/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{3 b^{5/4} \sqrt{a - b x^4}}$$

Result (type 4, 108 leaves):

$$\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x (-a + b x^4) - i a \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) / \left(3 \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} b \sqrt{a - b x^4} \right)$$

Problem 841: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a - b x^4}} dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{a^{1/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{b^{1/4} \sqrt{a - b x^4}}$$

Result (type 4, 72 leaves):

$$-\frac{i \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{a - b x^4}}$$

Problem 842: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a - b x^4}} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$-\frac{\sqrt{a - b x^4}}{3 a x^3} + \frac{b^{3/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{3 a^{3/4} \sqrt{a - b x^4}}$$

Result (type 4, 90 leaves):

$$-\frac{\frac{a}{x^3} + b x - \frac{i b \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-\sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{-\sqrt{b}}{\sqrt{a}}}}}{3 a \sqrt{a - b x^4}}$$

Problem 843: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 \sqrt{a - b x^4}} dx$$

Optimal (type 4, 102 leaves, 4 steps):

$$-\frac{\sqrt{a - b x^4}}{7 a x^7} - \frac{5 b \sqrt{a - b x^4}}{21 a^2 x^3} + \frac{5 b^{7/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{21 a^{7/4} \sqrt{a - b x^4}}$$

Result (type 4, 104 leaves):

$$-\frac{\frac{3 a^2}{x^7} - \frac{2 a b}{x^3} + 5 b^2 x - \frac{5 i b^2 \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-\sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{-\sqrt{b}}{\sqrt{a}}}}}{21 a^2 \sqrt{a - b x^4}}$$

Problem 844: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{\sqrt{a - b x^4}} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$-\frac{7 a x^3 \sqrt{a - b x^4}}{45 b^2} - \frac{x^7 \sqrt{a - b x^4}}{9 b} + \frac{7 a^{11/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{15 b^{11/4} \sqrt{a - b x^4}}$$

$$\frac{7 a^{11/4} \sqrt{1 - \frac{b x^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{15 b^{11/4} \sqrt{a - b x^4}}$$

Result (type 4, 134 leaves):

$$\frac{1}{45 b^2 \sqrt{a - b x^4}} \left((-a + b x^4) (7 a x^3 + 5 b x^7) + \frac{1}{\left(-\frac{\sqrt{b}}{\sqrt{a}}\right)^{3/2}} 21 i a^2 \sqrt{1 - \frac{b x^4}{a}} \right. \\ \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) \right)$$

Problem 845: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a - b x^4}} dx$$

Optimal (type 4, 135 leaves, 7 steps):

$$-\frac{x^3 \sqrt{a - b x^4}}{5 b} + \frac{3 a^{7/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{5 b^{7/4} \sqrt{a - b x^4}} - \\ \frac{3 a^{7/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{5 b^{7/4} \sqrt{a - b x^4}}$$

Result (type 4, 120 leaves):

$$\frac{1}{5 b \sqrt{a - b x^4}} \left(-a x^3 + b x^7 + \frac{1}{\left(-\frac{\sqrt{b}}{\sqrt{a}}\right)^{3/2}} 3 i a \sqrt{1 - \frac{b x^4}{a}} \right. \\ \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) \right)$$

Problem 846: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a - b x^4}} dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$\frac{a^{3/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{b^{3/4} \sqrt{a - b x^4}} - \frac{a^{3/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{b^{3/4} \sqrt{a - b x^4}}$$

Result (type 4, 100 leaves):

$$\left(i \sqrt{1 - \frac{b x^4}{a}} \left(\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x \right], -1 \right] - \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x \right], -1 \right] \right) \right) / \left(\left(-\frac{\sqrt{b}}{\sqrt{a}} \right)^{3/2} \sqrt{a - b x^4} \right)$$

Problem 847: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a - b x^4}} dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$-\frac{\sqrt{a - b x^4}}{a x} - \frac{b^{1/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{b^{1/4} x}{a^{1/4}} \right], -1 \right]}{a^{1/4} \sqrt{a - b x^4}} + \frac{b^{1/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{b^{1/4} x}{a^{1/4}} \right], -1 \right]}{a^{1/4} \sqrt{a - b x^4}}$$

Result (type 4, 115 leaves):

$$\frac{1}{\sqrt{a - b x^4}} \left(-\frac{1}{x} + \frac{b x^3}{a} - i \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{1 - \frac{b x^4}{a}} \left(\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x \right], -1 \right] - \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x \right], -1 \right] \right) \right)$$

Problem 848: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{a - b x^4}} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$-\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} - \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}x}{a^{1/4}}\right], -1\right]}{5a^{5/4}\sqrt{a-bx^4}} +$$

$$\frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}x}{a^{1/4}}\right], -1\right]}{5a^{5/4}\sqrt{a-bx^4}}$$

Result (type 4, 131 leaves):

$$\frac{1}{5a^2\sqrt{a-bx^4}} \left(\frac{(-a+bx^4)(a+3bx^4)}{x^5} - 3ia\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} b\sqrt{1-\frac{bx^4}{a}} \right.$$

$$\left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right], -1\right] \right) \right)$$

Problem 859: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}} dx$$

Optimal (type 4, 151 leaves, 4 steps):

$$-\frac{x^9}{2b\sqrt{a+bx^4}} - \frac{15ax\sqrt{a+bx^4}}{14b^3} + \frac{9x^5\sqrt{a+bx^4}}{14b^2} +$$

$$\frac{15a^{7/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{28b^{13/4}\sqrt{a+bx^4}}$$

Result (type 4, 106 leaves):

$$-15a^2x - 6abx^5 + 2b^2x^9 - \frac{15ia^2\sqrt{1+\frac{bx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}$$

$$\frac{}{14b^3\sqrt{a+bx^4}}$$

Problem 860: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{(a+bx^4)^{3/2}} dx$$

Optimal (type 4, 129 leaves, 3 steps):

$$-\frac{x^5}{2 b \sqrt{a+b x^4}} + \frac{5 x \sqrt{a+b x^4}}{6 b^2} - \frac{5 a^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{12 b^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 93 leaves):

$$\frac{5 a x + 2 b x^5 + \frac{5 i a \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{6 b^2 \sqrt{a+b x^4}}$$

Problem 861: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(a+b x^4)^{3/2}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$-\frac{x}{2 b \sqrt{a+b x^4}} + \frac{(\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} b^{5/4} \sqrt{a+b x^4}}$$

Result (type 4, 102 leaves):

$$-\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x + i \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{2 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

Problem 862: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+b x^4)^{3/2}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{x}{2 a \sqrt{a+b x^4}} + \frac{(\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} b^{1/4} \sqrt{a+b x^4}}$$

Result (type 4, 102 leaves):

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x - i \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{2 a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4}}$$

Problem 863: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 (a + b x^4)^{3/2}} dx$$

Optimal (type 4, 131 leaves, 3 steps):

$$\frac{1}{2 a x^3 \sqrt{a + b x^4}} - \frac{5 \sqrt{a + b x^4}}{6 a^2 x^3} - \frac{5 b^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{12 a^{9/4} \sqrt{a + b x^4}}$$

Result (type 4, 93 leaves):

$$\frac{-\frac{2 a}{x^3} - 5 b x + \frac{5 i b \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{6 a^2 \sqrt{a + b x^4}}$$

Problem 864: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 (a + b x^4)^{3/2}} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$\frac{1}{2 a x^7 \sqrt{a + b x^4}} - \frac{9 \sqrt{a + b x^4}}{14 a^2 x^7} + \frac{15 b \sqrt{a + b x^4}}{14 a^3 x^3} + \frac{15 b^{7/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{28 a^{13/4} \sqrt{a + b x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{2 a^2}{x^7} + \frac{6 a b}{x^3} + 15 b^2 x - \frac{15 i b^2 \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{14 a^3 \sqrt{a + b x^4}}$$

Problem 865: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{14}}{(a + b x^4)^{3/2}} dx$$

Optimal (type 4, 282 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{x^{11}}{2b\sqrt{a+bx^4}} - \frac{77ax^3\sqrt{a+bx^4}}{90b^3} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} + \frac{77a^2x\sqrt{a+bx^4}}{30b^{7/2}(\sqrt{a}+\sqrt{b}x^2)} - \\
 & \frac{77a^{9/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{30b^{15/4}\sqrt{a+bx^4}} + \\
 & \frac{77a^{9/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{60b^{15/4}\sqrt{a+bx^4}}
 \end{aligned}$$

Result (type 4, 183 leaves):

$$\begin{aligned}
 & \left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{b} x^3 (-77a^2 - 22abx^4 + 10b^2x^8) + \right. \\
 & 231a^{5/2} \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \\
 & \left. 231a^{5/2} \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) / \left(90 \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} b^{7/2} \sqrt{a+bx^4} \right)
 \end{aligned}$$

Problem 866: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}} dx$$

Optimal (type 4, 258 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{x^7}{2b\sqrt{a+bx^4}} + \frac{7x^3\sqrt{a+bx^4}}{10b^2} - \frac{21ax\sqrt{a+bx^4}}{10b^{5/2}(\sqrt{a}+\sqrt{b}x^2)} + \\
 & \frac{21a^{5/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{10b^{11/4}\sqrt{a+bx^4}} - \\
 & \frac{21a^{5/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{20b^{11/4}\sqrt{a+bx^4}}
 \end{aligned}$$

Result (type 4, 172 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{b} x^3 (7a + 2bx^4) - 21a^{3/2} \sqrt{1 + \frac{bx^4}{a}} \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] + \right. \\ \left. 21a^{3/2} \sqrt{1 + \frac{bx^4}{a}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) / \left(10 \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} b^{5/2} \sqrt{a + bx^4} \right)$$

Problem 867: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(a + bx^4)^{3/2}} dx$$

Optimal (type 4, 236 leaves, 4 steps):

$$-\frac{x^3}{2b\sqrt{a+bx^4}} + \frac{3x\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a} + \sqrt{b}x^2)} - \\ \frac{3a^{1/4}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{2b^{7/4}\sqrt{a+bx^4}} + \\ \frac{3a^{1/4}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{4b^{7/4}\sqrt{a+bx^4}}$$

Result (type 4, 163 leaves):

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{b} x^3 + 3\sqrt{a} \sqrt{1 + \frac{bx^4}{a}} \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \right. \\ \left. 3\sqrt{a} \sqrt{1 + \frac{bx^4}{a}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) / \left(2 \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} b^{3/2} \sqrt{a + bx^4} \right)$$

Problem 868: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(a + bx^4)^{3/2}} dx$$

Optimal (type 4, 239 leaves, 4 steps):

$$\frac{x^3}{2a\sqrt{a+bx^4}} - \frac{x\sqrt{a+bx^4}}{2a\sqrt{b}(\sqrt{a}+\sqrt{b}x^2)} +$$

$$\frac{(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} -$$

$$\frac{(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

Result (type 4, 163 leaves):

$$\left(i \left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{b}x^3 - \sqrt{a} \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] + \right. \right.$$

$$\left. \left. \sqrt{a} \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] \right) \right) / \left(2a^{3/2} \left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+bx^4} \right)$$

Problem 869: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}} dx$$

Optimal (type 4, 260 leaves, 5 steps):

$$\frac{1}{2ax\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{2a^2x} + \frac{3\sqrt{b}x\sqrt{a+bx^4}}{2a^2(\sqrt{a}+\sqrt{b}x^2)} -$$

$$\frac{3b^{1/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{7/4}\sqrt{a+bx^4}} +$$

$$\frac{3b^{1/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right], \frac{1}{2}\right]}{4a^{7/4}\sqrt{a+bx^4}}$$

Result (type 4, 178 leaves):

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} (2a+3bx^4) + 3\sqrt{a}\sqrt{b}x \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] - \right.$$

$$\left. 3\sqrt{a}\sqrt{b}x \sqrt{1 + \frac{bx^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right], -1\right] \right) / \left(2a^2 \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x \sqrt{a+bx^4} \right)$$

Problem 870: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 (a + b x^4)^{3/2}} dx$$

Optimal (type 4, 282 leaves, 6 steps):

$$\frac{1}{2 a x^5 \sqrt{a + b x^4}} - \frac{7 \sqrt{a + b x^4}}{10 a^2 x^5} + \frac{21 b \sqrt{a + b x^4}}{10 a^3 x} - \frac{21 b^{3/2} x \sqrt{a + b x^4}}{10 a^3 (\sqrt{a} + \sqrt{b} x^2)} +$$

$$\frac{21 b^{5/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{10 a^{11/4} \sqrt{a + b x^4}} -$$

$$\frac{21 b^{5/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{20 a^{11/4} \sqrt{a + b x^4}}$$

Result (type 4, 192 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} (-2 a^2 + 14 a b x^4 + 21 b^2 x^8) - \right.$$

$$21 \sqrt{a} b^{3/2} x^5 \sqrt{1 + \frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] + 21 \sqrt{a} b^{3/2} x^5$$

$$\left. \sqrt{1 + \frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] \right) / \left(10 a^3 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x^5 \sqrt{a + b x^4} \right)$$

Problem 871: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + b x^4)^{5/2}} dx$$

Optimal (type 4, 127 leaves, 3 steps):

$$\frac{x}{6 a (a + b x^4)^{3/2}} + \frac{5 x}{12 a^2 \sqrt{a + b x^4}} + \frac{5 (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{24 a^{9/4} b^{1/4} \sqrt{a + b x^4}}$$

Result (type 4, 99 leaves):

$$\frac{7 a x + 5 b x^5 - \frac{5 i (a+b x^4) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{12 a^2 (a+b x^4)^{3/2}}$$

Problem 925: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 74 leaves, 3 steps):

$$-\frac{5}{21} x \sqrt{1+x^4} + \frac{1}{7} x^5 \sqrt{1+x^4} + \frac{5 (1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{42 \sqrt{1+x^4}}$$

Result (type 4, 57 leaves):

$$-\frac{1}{21 \sqrt{1+x^4}} \left(5 x + 2 x^5 - 3 x^9 + 5 (-1)^{1/4} \sqrt{1+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 926: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{1+x^4} - \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{6 \sqrt{1+x^4}}$$

Result (type 4, 47 leaves):

$$\frac{x + x^5 + (-1)^{1/4} \sqrt{1+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right]}{3 \sqrt{1+x^4}}$$

Problem 927: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 43 leaves, 1 step):

$$\frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{2 \sqrt{1+x^4}}$$

Result (type 4, 21 leaves):

$$-(-1)^{1/4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right]$$

Problem 928: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{1+x^4}} dx$$

Optimal (type 4, 60 leaves, 2 steps):

$$-\frac{\sqrt{1+x^4}}{3x^3} - \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{6 \sqrt{1+x^4}}$$

Result (type 4, 55 leaves):

$$\frac{-1-x^4 + (-1)^{1/4} x^3 \sqrt{1+x^4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right]}{3x^3 \sqrt{1+x^4}}$$

Problem 929: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 \sqrt{1+x^4}} dx$$

Optimal (type 4, 76 leaves, 3 steps):

$$-\frac{\sqrt{1+x^4}}{7x^7} + \frac{5\sqrt{1+x^4}}{21x^3} + \frac{5(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{42 \sqrt{1+x^4}}$$

Result (type 4, 61 leaves):

$$\frac{1}{21x^7 \sqrt{1+x^4}} \left(-3 + 2x^4 + 5x^8 - 5(-1)^{1/4} x^7 \sqrt{1+x^4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 930: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$-\frac{7}{45} x^3 \sqrt{1+x^4} + \frac{1}{9} x^7 \sqrt{1+x^4} + \frac{7x \sqrt{1+x^4}}{15(1+x^2)} - \frac{7(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{15 \sqrt{1+x^4}} + \frac{7(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{30 \sqrt{1+x^4}}$$

Result (type 4, 72 leaves):

$$\frac{1}{45} \left(\frac{x^3 (-7 - 2x^4 + 5x^8)}{\sqrt{1+x^4}} - 21 (-1)^{3/4} \text{EllipticE}[\text{i ArcSinh}[(-1)^{1/4} x], -1] + 21 (-1)^{3/4} \text{EllipticF}[\text{i ArcSinh}[(-1)^{1/4} x], -1] \right)$$

Problem 931: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$\frac{1}{5} x^3 \sqrt{1+x^4} - \frac{3x \sqrt{1+x^4}}{5(1+x^2)} + \frac{3(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{5 \sqrt{1+x^4}} - \frac{3(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{10 \sqrt{1+x^4}}$$

Result (type 4, 73 leaves):

$$\frac{1}{5} \left(3 (-1)^{3/4} \text{EllipticE}[\text{i ArcSinh}[(-1)^{1/4} x], -1] + \frac{x^3 + x^7 - 3 (-1)^{3/4} \sqrt{1+x^4} \text{EllipticF}[\text{i ArcSinh}[(-1)^{1/4} x], -1]}{\sqrt{1+x^4}} \right)$$

Problem 932: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{1+x^4}} dx$$

Optimal (type 4, 103 leaves, 3 steps):

$$\frac{x \sqrt{1+x^4}}{1+x^2} - \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{1+x^4}} + \frac{(1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{2 \sqrt{1+x^4}}$$

Result (type 4, 37 leaves):

$$(-1)^{3/4} \left(-\text{EllipticE}[\text{i ArcSinh}[(-1)^{1/4} x], -1] + \text{EllipticF}[\text{i ArcSinh}[(-1)^{1/4} x], -1] \right)$$

Problem 933: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{1+x^4}} dx$$

Optimal (type 4, 117 leaves, 4 steps):

$$-\frac{\sqrt{1+x^4}}{x} + \frac{x\sqrt{1+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{2\sqrt{1+x^4}}$$

Result (type 4, 70 leaves):

$$-\frac{1}{x\sqrt{1+x^4}} - \frac{x^3}{\sqrt{1+x^4}} - (-1)^{3/4} \text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4}x\right], -1\right] + (-1)^{3/4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4}x\right], -1\right]$$

Problem 934: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \sqrt{1+x^4}} dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$-\frac{\sqrt{1+x^4}}{5x^5} + \frac{3\sqrt{1+x^4}}{5x} - \frac{3x\sqrt{1+x^4}}{5(1+x^2)} + \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{5\sqrt{1+x^4}} - \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{10\sqrt{1+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{5x^5\sqrt{1+x^4}} \left(-1 + 2x^4 + 3x^8 + 3(-1)^{3/4}x^5\sqrt{1+x^4} \text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4}x\right], -1\right] - 3(-1)^{3/4}x^5\sqrt{1+x^4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)$$

Problem 945: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{12}}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$-\frac{x^9}{2\sqrt{1+x^4}} - \frac{15}{14}x\sqrt{1+x^4} + \frac{9}{14}x^5\sqrt{1+x^4} + \frac{15(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{28\sqrt{1+x^4}}$$

Result (type 4, 57 leaves):

$$-\frac{1}{14\sqrt{1+x^4}} \left(15x + 6x^5 - 2x^9 + 15(-1)^{1/4}\sqrt{1+x^4} \operatorname{EllipticF}\left[\operatorname{i}\operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)$$

Problem 946: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 74 leaves, 3 steps):

$$-\frac{x^5}{2\sqrt{1+x^4}} + \frac{5}{6}x\sqrt{1+x^4} - \frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{12\sqrt{1+x^4}}$$

Result (type 4, 52 leaves):

$$\frac{5x + 2x^5 + 5(-1)^{1/4}\sqrt{1+x^4} \operatorname{EllipticF}\left[\operatorname{i}\operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right]}{6\sqrt{1+x^4}}$$

Problem 947: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$-\frac{x}{2\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 38 leaves):

$$-\frac{x}{2\sqrt{1+x^4}} - \frac{1}{2}(-1)^{1/4} \operatorname{EllipticF}\left[\operatorname{i}\operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right]$$

Problem 948: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$\frac{x}{2\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 37 leaves):

$$\frac{1}{2} \left(\frac{x}{\sqrt{1+x^4}} - (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 949: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 (1+x^4)^{3/2}} dx$$

Optimal (type 4, 76 leaves, 3 steps):

$$\frac{1}{2x^3\sqrt{1+x^4}} - \frac{5\sqrt{1+x^4}}{6x^3} - \frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{12\sqrt{1+x^4}}$$

Result (type 4, 46 leaves):

$$\frac{1}{6} \left(\frac{-2-5x^4}{x^3\sqrt{1+x^4}} + 5(-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 950: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 (1+x^4)^{3/2}} dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{1}{2x^7\sqrt{1+x^4}} - \frac{9\sqrt{1+x^4}}{14x^7} + \frac{15\sqrt{1+x^4}}{14x^3} + \frac{15(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{28\sqrt{1+x^4}}$$

Result (type 4, 61 leaves):

$$\frac{1}{14x^7\sqrt{1+x^4}} \left(-2 + 6x^4 + 15x^8 - 15(-1)^{1/4} x^7 \sqrt{1+x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 951: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{14}}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{x^{11}}{2\sqrt{1+x^4}} - \frac{77}{90}x^3\sqrt{1+x^4} + \frac{11}{18}x^7\sqrt{1+x^4} + \\
 & \frac{77x\sqrt{1+x^4}}{30(1+x^2)} - \frac{77(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{30\sqrt{1+x^4}} + \\
 & \frac{77(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{60\sqrt{1+x^4}}
 \end{aligned}$$

Result (type 4, 72 leaves):

$$\begin{aligned}
 & \frac{1}{90} \left(\frac{x^3(-77 - 22x^4 + 10x^8)}{\sqrt{1+x^4}} - 231(-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] + \right. \\
 & \left. 231(-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)
 \end{aligned}$$

Problem 952: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{x^7}{2\sqrt{1+x^4}} + \frac{7}{10}x^3\sqrt{1+x^4} - \frac{21x\sqrt{1+x^4}}{10(1+x^2)} + \frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{10\sqrt{1+x^4}} - \\
 & \frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}[x], \frac{1}{2}\right]}{20\sqrt{1+x^4}}
 \end{aligned}$$

Result (type 4, 75 leaves):

$$\begin{aligned}
 & \frac{1}{10} \left(\frac{7x^3}{\sqrt{1+x^4}} + \frac{2x^7}{\sqrt{1+x^4}} + 21(-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] - \right. \\
 & \left. 21(-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)
 \end{aligned}$$

Problem 953: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$-\frac{x^3}{2\sqrt{1+x^4}} + \frac{3x\sqrt{1+x^4}}{2(1+x^2)} - \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}[x], \frac{1}{2}\right]}{2\sqrt{1+x^4}} +$$

$$\frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 61 leaves):

$$\frac{1}{2} \left(-\frac{x^3}{\sqrt{1+x^4}} - 3(-1)^{3/4} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] + \right.$$

$$\left. 3(-1)^{3/4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)$$

Problem 954: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(1+x^4)^{3/2}} dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$\frac{x^3}{2\sqrt{1+x^4}} - \frac{x\sqrt{1+x^4}}{2(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}[x], \frac{1}{2}\right]}{2\sqrt{1+x^4}} -$$

$$\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2\text{ArcTan}[x], \frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 59 leaves):

$$\frac{1}{2} \left(\frac{x^3}{\sqrt{1+x^4}} + (-1)^{3/4} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] - \right.$$

$$\left. (-1)^{3/4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4}x\right], -1\right] \right)$$

Problem 955: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2(1+x^4)^{3/2}} dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$\frac{1}{2 x \sqrt{1+x^4}} - \frac{3 \sqrt{1+x^4}}{2 x} + \frac{3 x \sqrt{1+x^4}}{2 (1+x^2)} - \frac{3 (1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{2 \sqrt{1+x^4}} +$$

$$\frac{3 (1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{4 \sqrt{1+x^4}}$$

Result (type 4, 75 leaves):

$$\frac{1}{2} \left(-\frac{2}{x \sqrt{1+x^4}} - \frac{3 x^3}{\sqrt{1+x^4}} - 3 (-1)^{3/4} \text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] + \right.$$

$$\left. 3 (-1)^{3/4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 956: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$\frac{1}{2 x^5 \sqrt{1+x^4}} - \frac{7 \sqrt{1+x^4}}{10 x^5} + \frac{21 \sqrt{1+x^4}}{10 x} -$$

$$\frac{21 x \sqrt{1+x^4}}{10 (1+x^2)} + \frac{21 (1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{10 \sqrt{1+x^4}} -$$

$$\frac{21 (1+x^2) \sqrt{\frac{1+x^4}{(1+x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{20 \sqrt{1+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{10 x^5 \sqrt{1+x^4}} \left(-2 + 14 x^4 + 21 x^8 + 21 (-1)^{3/4} x^5 \sqrt{1+x^4} \text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] - \right.$$

$$\left. 21 (-1)^{3/4} x^5 \sqrt{1+x^4} \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} x\right], -1\right] \right)$$

Problem 957: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+x^4)^{5/2}} dx$$

Optimal (type 4, 72 leaves, 3 steps):

$$\frac{x}{6(1+x^4)^{3/2}} + \frac{5x}{12\sqrt{1+x^4}} + \frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}[x], \frac{1}{2}\right]}{24\sqrt{1+x^4}}$$

Result (type 4, 52 leaves):

$$\frac{7x + 5x^5 - 5(-1)^{1/4}(1+x^4)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4}x\right], -1\right]}{12(1+x^4)^{3/2}}$$

Problem 972: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-4+x^4}} dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{2} \operatorname{ArcTanh}\left[\frac{x^2}{\sqrt{-4+x^4}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{4} \operatorname{Log}\left[1 - \frac{x^2}{\sqrt{-4+x^4}}\right] + \frac{1}{4} \operatorname{Log}\left[1 + \frac{x^2}{\sqrt{-4+x^4}}\right]$$

Problem 982: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{3-bx^4}} dx$$

Optimal (type 4, 54 leaves, 4 steps):

$$\frac{3^{1/4} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}x}{3^{1/4}}\right], -1\right]}{b^{3/4}} - \frac{3^{1/4} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}x}{3^{1/4}}\right], -1\right]}{b^{3/4}}$$

Result (type 4, 76 leaves):

$$\frac{1}{b} i 3^{1/4} \sqrt{-\sqrt{b}} \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\sqrt{b}}x}{3^{1/4}}\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\sqrt{b}}x}{3^{1/4}}\right], -1\right] \right)$$

Problem 991: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^4)^{1/4}}{x} dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$(a+bx^4)^{1/4} - \frac{1}{2} a^{1/4} \operatorname{ArcTan}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right] - \frac{1}{2} a^{1/4} \operatorname{ArcTanh}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 61 leaves):

$$\frac{3 (a + b x^4) - a \left(1 + \frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4}\right]}{3 (a + b x^4)^{3/4}}$$

Problem 992: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^5} dx$$

Optimal (type 3, 75 leaves, 6 steps):

$$-\frac{(a + b x^4)^{1/4}}{4 x^4} - \frac{b \text{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{3/4}} - \frac{b \text{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{-3 (a + b x^4) - b \left(1 + \frac{a}{b x^4}\right)^{3/4} x^4 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4}\right]}{12 x^4 (a + b x^4)^{3/4}}$$

Problem 993: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^9} dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$-\frac{(a + b x^4)^{1/4}}{8 x^8} - \frac{b (a + b x^4)^{1/4}}{32 a x^4} + \frac{3 b^2 \text{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}} + \frac{3 b^2 \text{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}}$$

Result (type 5, 82 leaves):

$$\frac{\left(-4 a^2 - 5 a b x^4 - b^2 x^8 + b^2 \left(1 + \frac{a}{b x^4}\right)^{3/4} x^8 \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4}\right]\right)}{\left(32 a x^8 (a + b x^4)^{3/4}\right)}$$

Problem 994: Result unnecessarily involves higher level functions.

$$\int x^9 (a + b x^4)^{1/4} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{2 a^2 x^2 (a + b x^4)^{1/4}}{77 b^2} + \frac{a x^6 (a + b x^4)^{1/4}}{77 b} + \frac{\frac{1}{11} x^{10} (a + b x^4)^{1/4} + \frac{4 a^{7/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{77 b^{5/2} (a + b x^4)^{3/4}}}{77 b^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{1}{77 b^2 (a + b x^4)^{3/4}} x^2 \left(-2 a^3 - a^2 b x^4 + 8 a b^2 x^8 + 7 b^3 x^{12} + 2 a^3 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right)$$

Problem 995: Result unnecessarily involves higher level functions.

$$\int x^5 (a + b x^4)^{1/4} dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$\frac{a x^2 (a + b x^4)^{1/4}}{21 b} + \frac{1}{7} x^6 (a + b x^4)^{1/4} - \frac{2 a^{5/2} \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{21 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 78 leaves):

$$\frac{1}{21 b (a + b x^4)^{3/4}} x^2 \left(a^2 + 4 a b x^4 + 3 b^2 x^8 - a^2 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right)$$

Problem 996: Result unnecessarily involves higher level functions.

$$\int x (a + b x^4)^{1/4} dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$\frac{1}{3} x^2 (a + b x^4)^{1/4} + \frac{a^{3/2} \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{3 \sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 63 leaves):

$$\frac{x^2 \left(2 (a + b x^4) + a \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right)}{6 (a + b x^4)^{3/4}}$$

Problem 997: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^3} dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$-\frac{(a + b x^4)^{1/4}}{2 x^2} + \frac{\sqrt{a} \sqrt{b} \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{2 (a + b x^4)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{-2 (a + b x^4) + b x^4 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{4 x^2 (a + b x^4)^{3/4}}$$

Problem 998: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^7} dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$-\frac{(a + b x^4)^{1/4}}{6 x^6} - \frac{b (a + b x^4)^{1/4}}{12 a x^2} - \frac{b^{3/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 \sqrt{a} (a + b x^4)^{3/4}}$$

Result (type 5, 85 leaves):

$$\frac{\left(-2 (2 a^2 + 3 a b x^4 + b^2 x^8) - b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{(24 a x^6 (a + b x^4)^{3/4})}$$

Problem 999: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^{11}} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{(a + b x^4)^{1/4}}{10 x^{10}} - \frac{b (a + b x^4)^{1/4}}{60 a x^6} + \frac{b^2 (a + b x^4)^{1/4}}{24 a^2 x^2} + \frac{b^{5/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{24 a^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 94 leaves):

$$\frac{\left(-24 a^3 - 28 a^2 b x^4 + 6 a b^2 x^8 + 10 b^3 x^{12} + 5 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{(240 a^2 x^{10} (a + b x^4)^{3/4})}$$

Problem 1000: Result unnecessarily involves higher level functions.

$$\int x^6 (a + b x^4)^{1/4} dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{a x^3 (a + b x^4)^{1/4}}{32 b} + \frac{1}{8} x^7 (a + b x^4)^{1/4} + \frac{3 a^2 \text{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{64 b^{7/4}} - \frac{3 a^2 \text{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{64 b^{7/4}}$$

Result (type 5, 78 leaves):

$$\frac{1}{32 b (a + b x^4)^{3/4}} x^3 \left(a^2 + 5 a b x^4 + 4 b^2 x^8 - a^2 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right)$$

Problem 1001: Result unnecessarily involves higher level functions.

$$\int x^2 (a + b x^4)^{1/4} dx$$

Optimal (type 3, 77 leaves, 5 steps):

$$\frac{1}{4} x^3 (a + b x^4)^{1/4} - \frac{a \text{ArcTan} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{8 b^{3/4}} + \frac{a \text{ArcTanh} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{8 b^{3/4}}$$

Result (type 5, 63 leaves):

$$\frac{x^3 \left(3 (a + b x^4) + a \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right)}{12 (a + b x^4)^{3/4}}$$

Problem 1002: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^2} dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$-\frac{(a + b x^4)^{1/4}}{x} - \frac{1}{2} b^{1/4} \text{ArcTan} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right] + \frac{1}{2} b^{1/4} \text{ArcTanh} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]$$

Result (type 5, 66 leaves):

$$\frac{-3 (a + b x^4) + b x^4 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right]}{3 x (a + b x^4)^{3/4}}$$

Problem 1007: Result unnecessarily involves higher level functions.

$$\int x^{12} (a + b x^4)^{1/4} dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$\frac{3 a^3 x (a + b x^4)^{1/4}}{112 b^3} - \frac{3 a^2 x^5 (a + b x^4)^{1/4}}{280 b^2} + \frac{a x^9 (a + b x^4)^{1/4}}{140 b} + \frac{1}{14} x^{13} (a + b x^4)^{1/4} + \frac{3 a^{7/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{112 b^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 101 leaves):

$$\left(15 a^4 x + 9 a^3 b x^5 - 2 a^2 b^2 x^9 + 44 a b^3 x^{13} + 40 b^4 x^{17} - 15 a^4 x \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right) / \left(560 b^3 (a + b x^4)^{3/4} \right)$$

Problem 1008: Result unnecessarily involves higher level functions.

$$\int x^8 (a + b x^4)^{1/4} dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$-\frac{a^2 x (a + b x^4)^{1/4}}{24 b^2} + \frac{a x^5 (a + b x^4)^{1/4}}{60 b} + \frac{1}{10} x^9 (a + b x^4)^{1/4} - \frac{a^{5/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{24 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 90 leaves):

$$\left(-5 a^3 x - 3 a^2 b x^5 + 14 a b^2 x^9 + 12 b^3 x^{13} + 5 a^3 x \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right) / \left(120 b^2 (a + b x^4)^{3/4} \right)$$

Problem 1009: Result unnecessarily involves higher level functions.

$$\int x^4 (a + b x^4)^{1/4} dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{a x (a + b x^4)^{1/4}}{12 b} + \frac{1}{6} x^5 (a + b x^4)^{1/4} + \frac{a^{3/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{12 \sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 76 leaves):

$$\frac{1}{12 b (a + b x^4)^{3/4}} x \left(a^2 + 3 a b x^4 + 2 b^2 x^8 - a^2 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)$$

Problem 1010: Result unnecessarily involves higher level functions.

$$\int (a + b x^4)^{1/4} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{1}{2} x (a + b x^4)^{1/4} - \frac{\sqrt{a} \sqrt{b} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{2 (a + b x^4)^{3/4}}$$

Result (type 5, 58 leaves):

$$\frac{x \left(a + b x^4 + a \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)}{2 \left(a + b x^4 \right)^{3/4}}$$

Problem 1011: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^4} dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$-\frac{(a + b x^4)^{1/4}}{3 x^3} - \frac{b^{3/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{3 \sqrt{a} \left(a + b x^4 \right)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{-a - b x^4 + b x^4 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right]}{3 x^3 \left(a + b x^4 \right)^{3/4}}$$

Problem 1012: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^8} dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$-\frac{(a + b x^4)^{1/4}}{7 x^7} - \frac{b (a + b x^4)^{1/4}}{21 a x^3} + \frac{2 b^{5/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{21 a^{3/2} \left(a + b x^4 \right)^{3/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-3 a^2 - 4 a b x^4 - b^2 x^8 - 2 b^2 x^8 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)}{\left(21 a x^7 \left(a + b x^4 \right)^{3/4} \right)}$$

Problem 1013: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{1/4}}{x^{12}} dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$\frac{-\frac{(a+bx^4)^{1/4}}{11x^{11}} - \frac{b(a+bx^4)^{1/4}}{77ax^7} + \frac{2b^2(a+bx^4)^{1/4}}{77a^2x^3} - 4b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCot}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{77a^{5/2}(a+bx^4)^{3/4}}$$

Result (type 5, 93 leaves):

$$\frac{\left(-7a^3 - 8a^2bx^4 + a^2b^2x^8 + 2b^3x^{12} + 4b^3x^{12}\left(1+\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{bx^4}{a}\right]\right)}{\left(77a^2x^{11}(a+bx^4)^{3/4}\right)}$$

Problem 1014: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^4)^{1/4}}{x^{16}} dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$\frac{-\frac{(a+bx^4)^{1/4}}{15x^{15}} - \frac{b(a+bx^4)^{1/4}}{165ax^{11}} + \frac{2b^2(a+bx^4)^{1/4}}{231a^2x^7} - \frac{4b^3(a+bx^4)^{1/4}}{231a^3x^3} + \frac{8b^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCot}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{231a^{7/2}(a+bx^4)^{3/4}}}{}$$

Result (type 5, 105 leaves):

$$\frac{\left(-77a^4 - 84a^3bx^4 + 3a^2b^2x^8 - 10a^2b^3x^{12} - 20b^4x^{16} - 40b^4x^{16}\left(1+\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{bx^4}{a}\right]\right)}{\left(1155a^3x^{15}(a+bx^4)^{3/4}\right)}$$

Problem 1020: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^4)^{3/4}}{x} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{1}{3}(a+bx^4)^{3/4} + \frac{1}{2}a^{3/4} \operatorname{ArcTan}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right] - \frac{1}{2}a^{3/4} \operatorname{ArcTanh}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 58 leaves):

$$\frac{a+bx^4 - 3a\left(1+\frac{a}{bx^4}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{bx^4}\right]}{3(a+bx^4)^{1/4}}$$

Problem 1021: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^5} dx$$

Optimal (type 3, 75 leaves, 6 steps):

$$-\frac{(a + b x^4)^{3/4}}{4 x^4} + \frac{3 b \operatorname{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{1/4}} - \frac{3 b \operatorname{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{1/4}}$$

Result (type 5, 67 leaves):

$$\frac{-a - b x^4 - 3 b \left(1 + \frac{a}{b x^4}\right)^{1/4} x^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4}\right]}{4 x^4 (a + b x^4)^{1/4}}$$

Problem 1022: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^9} dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$-\frac{(a + b x^4)^{3/4}}{8 x^8} - \frac{3 b (a + b x^4)^{3/4}}{32 a x^4} - \frac{3 b^2 \operatorname{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{5/4}} + \frac{3 b^2 \operatorname{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{5/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-4 a^2 - 7 a b x^4 - 3 b^2 x^8 + 3 b^2 \left(1 + \frac{a}{b x^4}\right)^{1/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4}\right]\right)}{\left(32 a x^8 (a + b x^4)^{1/4}\right)}$$

Problem 1023: Result unnecessarily involves higher level functions.

$$\int x^9 (a + b x^4)^{3/4} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$\frac{4 a^3 x^2}{65 b^2 (a + b x^4)^{1/4}} - \frac{2 a^2 x^2 (a + b x^4)^{3/4}}{65 b^2} + \frac{a x^6 (a + b x^4)^{3/4}}{39 b} + \frac{1}{13} x^{10} (a + b x^4)^{3/4} - \frac{4 a^{7/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{65 b^{5/2} (a + b x^4)^{1/4}}$$

Result (type 5, 91 leaves):

$$\left(x^2 \left(-6 a^3 - a^2 b x^4 + 20 a b^2 x^8 + 15 b^3 x^{12} + 6 a^3 \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right) \right) / \left(195 b^2 (a + b x^4)^{1/4} \right)$$

Problem 1024: Result unnecessarily involves higher level functions.

$$\int x^5 (a + b x^4)^{3/4} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{2 a^2 x^2}{15 b (a + b x^4)^{1/4}} + \frac{a x^2 (a + b x^4)^{3/4}}{15 b} + \frac{1}{9} x^6 (a + b x^4)^{3/4} + \frac{2 a^{5/2} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{15 b^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{45 b (a + b x^4)^{1/4}} x^2 \left(3 a^2 + 8 a b x^4 + 5 b^2 x^8 - 3 a^2 \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right)$$

Problem 1025: Result unnecessarily involves higher level functions.

$$\int x (a + b x^4)^{3/4} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{3 a x^2}{5 (a + b x^4)^{1/4}} + \frac{1}{5} x^2 (a + b x^4)^{3/4} - \frac{3 a^{3/2} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{5 \sqrt{b} (a + b x^4)^{1/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(2 (a + b x^4) + 3 a \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right)}{10 (a + b x^4)^{1/4}}$$

Problem 1026: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^3} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{3 b x^2}{2 (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{2 x^2} - \frac{3 \sqrt{a} \sqrt{b} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{2 (a + b x^4)^{1/4}}$$

Result (type 5, 67 leaves):

$$\frac{-2 (a + b x^4) + 3 b x^4 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{4 x^2 (a + b x^4)^{1/4}}$$

Problem 1027: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^7} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{b^2 x^2}{4 a (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{6 x^6} - \frac{b (a + b x^4)^{3/4}}{4 a x^2} - \frac{b^{3/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 \sqrt{a} (a + b x^4)^{1/4}}$$

Result (type 5, 86 leaves):

$$\frac{\left(-2 (2 a^2 + 5 a b x^4 + 3 b^2 x^8) + 3 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{(24 a x^6 (a + b x^4)^{1/4})}$$

Problem 1028: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^{11}} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$\frac{3 b^3 x^2}{40 a^2 (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{10 x^{10}} - \frac{b (a + b x^4)^{3/4}}{20 a x^6} + \frac{3 b^2 (a + b x^4)^{3/4}}{40 a^2 x^2} + \frac{3 b^{5/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 a^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{\left(-8 a^3 - 12 a^2 b x^4 + 2 a b^2 x^8 + 6 b^3 x^{12} - 3 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{(80 a^2 x^{10} (a + b x^4)^{1/4})}$$

Problem 1038: Result unnecessarily involves higher level functions.

$$\int x^{10} (a + b x^4)^{3/4} dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$\frac{3 a^3 x^3}{80 b^2 (a + b x^4)^{1/4}} - \frac{a^2 x^3 (a + b x^4)^{3/4}}{40 b^2} + \frac{3 a x^7 (a + b x^4)^{3/4}}{140 b} +$$

$$\frac{1}{14} x^{11} (a + b x^4)^{3/4} + \frac{3 a^{7/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{80 b^{5/2} (a + b x^4)^{1/4}}$$

Result (type 5, 91 leaves):

$$\left(x^3 \left(-7 a^3 - a^2 b x^4 + 26 a b^2 x^8 + 20 b^3 x^{12} + \right. \right.$$

$$\left. \left. 7 a^3 \left(1 + \frac{b x^4}{a} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right) \right) / \left(280 b^2 (a + b x^4)^{1/4} \right)$$

Problem 1039: Result unnecessarily involves higher level functions.

$$\int x^6 (a + b x^4)^{3/4} dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$-\frac{3 a^2 x^3}{40 b (a + b x^4)^{1/4}} + \frac{a x^3 (a + b x^4)^{3/4}}{20 b} + \frac{1}{10} x^7 (a + b x^4)^{3/4} -$$

$$\frac{3 a^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 b^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 78 leaves):

$$\frac{1}{20 b (a + b x^4)^{1/4}} x^3 \left(a^2 + 3 a b x^4 + 2 b^2 x^8 - a^2 \left(1 + \frac{b x^4}{a} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1040: Result unnecessarily involves higher level functions.

$$\int x^2 (a + b x^4)^{3/4} dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$\frac{a x^3}{4 (a + b x^4)^{1/4}} + \frac{1}{6} x^3 (a + b x^4)^{3/4} + \frac{a^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 \sqrt{b} (a + b x^4)^{1/4}}$$

Result (type 5, 60 leaves):

$$\frac{x^3 \left(a + b x^4 + a \left(1 + \frac{b x^4}{a} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)}{6 (a + b x^4)^{1/4}}$$

Problem 1041: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^2} dx$$

Optimal (type 4, 97 leaves, 6 steps):

$$\frac{3 b x^3}{2 (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{x} + \frac{3 \sqrt{a} \sqrt{b} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 (a + b x^4)^{1/4}}$$

Result (type 5, 63 leaves):

$$\frac{-a - b x^4 + b x^4 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]}{x (a + b x^4)^{1/4}}$$

Problem 1042: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^6} dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$-\frac{3 b}{5 x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{5 x^5} + \frac{3 b^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{5 \sqrt{a} (a + b x^4)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-a^2 - 4 a b x^4 - 3 b^2 x^8 + 2 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)}{\left(5 a x^5 (a + b x^4)^{1/4}\right)}$$

Problem 1043: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^{10}} dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$\frac{2 b^2}{15 a x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{9 x^9} - \frac{b (a + b x^4)^{3/4}}{15 a x^5} - \frac{2 b^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{15 a^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{\left(-5 a^3 - 8 a^2 b x^4 + 3 a b^2 x^8 + 6 b^3 x^{12} - 4 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)}{\left(45 a^2 x^9 (a + b x^4)^{1/4}\right)}$$

Problem 1044: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{3/4}}{x^{14}} dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$\begin{aligned} & -\frac{4 b^3}{65 a^2 x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{13 x^{13}} - \frac{b (a + b x^4)^{3/4}}{39 a x^9} + \\ & \frac{2 b^2 (a + b x^4)^{3/4}}{65 a^2 x^5} + \frac{4 b^{7/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{65 a^{5/2} (a + b x^4)^{1/4}} \end{aligned}$$

Result (type 5, 104 leaves):

$$\begin{aligned} & \left(-15 a^4 - 20 a^3 b x^4 + a^2 b^2 x^8 - 6 a b^3 x^{12} - 12 b^4 x^{16} + \right. \\ & \left. 8 b^4 x^{16} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right) / \left(195 a^3 x^{13} (a + b x^4)^{1/4}\right) \end{aligned}$$

Problem 1050: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x} dx$$

Optimal (type 3, 83 leaves, 7 steps):

$$a (a + b x^4)^{1/4} + \frac{1}{5} (a + b x^4)^{5/4} - \frac{1}{2} a^{5/4} \text{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right] - \frac{1}{2} a^{5/4} \text{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 76 leaves):

$$\frac{1}{15 (a + b x^4)^{3/4}} \left(3 (6 a^2 + 7 a b x^4 + b^2 x^8) - 5 a^2 \left(1 + \frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4}\right]\right)$$

Problem 1051: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^5} dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{5}{4} b (a + b x^4)^{1/4} - \frac{(a + b x^4)^{5/4}}{4 x^4} - \frac{5}{8} a^{1/4} b \text{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right] - \frac{5}{8} a^{1/4} b \text{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 73 leaves):

$$\left(b - \frac{a}{4x^4}\right) (a + bx^4)^{1/4} - \frac{5ab \left(1 + \frac{a}{bx^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{bx^4}\right]}{12 (a + bx^4)^{3/4}}$$

Problem 1052: Result unnecessarily involves higher level functions.

$$\int \frac{(a + bx^4)^{5/4}}{x^9} dx$$

Optimal (type 3, 98 leaves, 7 steps):

$$-\frac{5b(a + bx^4)^{1/4}}{32x^4} - \frac{(a + bx^4)^{5/4}}{8x^8} - \frac{5b^2 \text{ArcTan}\left[\frac{(a + bx^4)^{1/4}}{a^{1/4}}\right]}{64a^{3/4}} - \frac{5b^2 \text{ArcTanh}\left[\frac{(a + bx^4)^{1/4}}{a^{1/4}}\right]}{64a^{3/4}}$$

Result (type 5, 85 leaves):

$$\left(-\frac{a}{8x^8} - \frac{9b}{32x^4}\right) (a + bx^4)^{1/4} - \frac{5b^2 \left(\frac{a + bx^4}{bx^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{bx^4}\right]}{96 (a + bx^4)^{3/4}}$$

Problem 1053: Result unnecessarily involves higher level functions.

$$\int x^9 (a + bx^4)^{5/4} dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{2a^3x^2(a + bx^4)^{1/4}}{231b^2} + \frac{a^2x^6(a + bx^4)^{1/4}}{231b} + \frac{1}{33}ax^{10}(a + bx^4)^{1/4} + \frac{1}{15}x^{10}(a + bx^4)^{5/4} + \frac{4a^{9/2}\left(1 + \frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{231b^{5/2}(a + bx^4)^{3/4}}$$

Result (type 5, 102 leaves):

$$\left(x^2 \left(-10a^4 - 5a^3bx^4 + 117a^2b^2x^8 + 189ab^3x^{12} + 77b^4x^{16} + 10a^4 \left(1 + \frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{bx^4}{a}\right]\right)\right) / \left(1155b^2(a + bx^4)^{3/4}\right)$$

Problem 1054: Result unnecessarily involves higher level functions.

$$\int x^5 (a + bx^4)^{5/4} dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$\frac{5 a^2 x^2 (a + b x^4)^{1/4}}{231 b} + \frac{5}{77} a x^6 (a + b x^4)^{1/4} + \frac{1}{11} x^6 (a + b x^4)^{5/4} - \frac{10 a^{7/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{231 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{1}{231 b (a + b x^4)^{3/4}} x^2 \left(5 a^3 + 41 a^2 b x^4 + 57 a b^2 x^8 + 21 b^3 x^{12} - 5 a^3 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right] \right)$$

Problem 1055: Result unnecessarily involves higher level functions.

$$\int x (a + b x^4)^{5/4} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{5}{21} a x^2 (a + b x^4)^{1/4} + \frac{1}{7} x^2 (a + b x^4)^{5/4} + \frac{5 a^{5/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{21 \sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 77 leaves):

$$\frac{1}{42 (a + b x^4)^{3/4}} x^2 \left(16 a^2 + 22 a b x^4 + 6 b^2 x^8 + 5 a^2 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right] \right)$$

Problem 1056: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^3} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{5}{6} b x^2 (a + b x^4)^{1/4} - \frac{(a + b x^4)^{5/4}}{2 x^2} + \frac{5 a^{3/2} \sqrt{b} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{6 (a + b x^4)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{12 x^2 (a + b x^4)^{3/4}} \left(-6 a^2 - 2 a b x^4 + 4 b^2 x^8 + 5 a b x^4 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right] \right)$$

Problem 1057: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^7} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$-\frac{5 b (a + b x^4)^{1/4}}{12 x^2} - \frac{(a + b x^4)^{5/4}}{6 x^6} + \frac{5 \sqrt{a} b^{3/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 (a + b x^4)^{3/4}}$$

Result (type 5, 85 leaves):

$$\left(-\frac{a}{6 x^6} - \frac{7 b}{12 x^2}\right) (a + b x^4)^{1/4} + \frac{5 b^2 x^2 \left(\frac{a + b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{24 (a + b x^4)^{3/4}}$$

Problem 1058: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^{11}} dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{b (a + b x^4)^{1/4}}{12 x^6} - \frac{b^2 (a + b x^4)^{1/4}}{24 a x^2} - \frac{(a + b x^4)^{5/4}}{10 x^{10}} - \frac{b^{5/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{24 \sqrt{a} (a + b x^4)^{3/4}}$$

Result (type 5, 97 leaves):

$$\left(-2 (12 a^3 + 34 a^2 b x^4 + 27 a b^2 x^8 + 5 b^3 x^{12}) - 5 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / (240 a x^{10} (a + b x^4)^{3/4})$$

Problem 1059: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^{15}} dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{b (a + b x^4)^{1/4}}{28 x^{10}} - \frac{b^2 (a + b x^4)^{1/4}}{168 a x^6} + \frac{5 b^3 (a + b x^4)^{1/4}}{336 a^2 x^2} - \frac{(a + b x^4)^{5/4}}{14 x^{14}} + \frac{5 b^{7/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{336 a^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 105 leaves):

$$\left(-48 a^4 - 120 a^3 b x^4 - 76 a^2 b^2 x^8 + 6 a b^3 x^{12} + 10 b^4 x^{16} + 5 b^4 x^{16} \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a} \right] \right) / \left(672 a^2 x^{14} (a + b x^4)^{3/4} \right)$$

Problem 1060: Result unnecessarily involves higher level functions.

$$\int x^{10} (a + b x^4)^{5/4} dx$$

Optimal (type 3, 148 leaves, 8 steps):

$$-\frac{35 a^3 x^3 (a + b x^4)^{1/4}}{6144 b^2} + \frac{5 a^2 x^7 (a + b x^4)^{1/4}}{1536 b} + \frac{5}{192} a x^{11} (a + b x^4)^{1/4} + \frac{1}{16} x^{11} (a + b x^4)^{5/4} - \frac{35 a^4 \text{ArcTan} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{4096 b^{11/4}} + \frac{35 a^4 \text{ArcTanh} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{4096 b^{11/4}}$$

Result (type 5, 102 leaves):

$$\left(x^3 \left(-35 a^4 - 15 a^3 b x^4 + 564 a^2 b^2 x^8 + 928 a b^3 x^{12} + 384 b^4 x^{16} + 35 a^4 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right) \right) / \left(6144 b^2 (a + b x^4)^{3/4} \right)$$

Problem 1061: Result unnecessarily involves higher level functions.

$$\int x^6 (a + b x^4)^{5/4} dx$$

Optimal (type 3, 124 leaves, 7 steps):

$$\frac{5 a^2 x^3 (a + b x^4)^{1/4}}{384 b} + \frac{5}{96} a x^7 (a + b x^4)^{1/4} + \frac{1}{12} x^7 (a + b x^4)^{5/4} + \frac{5 a^3 \text{ArcTan} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{256 b^{7/4}} - \frac{5 a^3 \text{ArcTanh} \left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}} \right]}{256 b^{7/4}}$$

Result (type 5, 91 leaves):

$$\frac{1}{384 b (a + b x^4)^{3/4}} x^3 \left(5 a^3 + 57 a^2 b x^4 + 84 a b^2 x^8 + 32 b^3 x^{12} - 5 a^3 \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right)$$

Problem 1062: Result unnecessarily involves higher level functions.

$$\int x^2 (a + b x^4)^{5/4} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$\frac{5}{32} a x^3 (a + b x^4)^{1/4} + \frac{1}{8} x^3 (a + b x^4)^{5/4} - \frac{5 a^2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{64 b^{3/4}} + \frac{5 a^2 \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{64 b^{3/4}}$$

Result (type 5, 77 leaves):

$$\frac{1}{96 (a + b x^4)^{3/4}} x^3 \left(27 a^2 + 39 a b x^4 + 12 b^2 x^8 + 5 a^2 \left(1 + \frac{b x^4}{a} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1063: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^2} dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$\frac{5}{4} b x^3 (a + b x^4)^{1/4} - \frac{(a + b x^4)^{5/4}}{x} - \frac{5}{8} a b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right] + \frac{5}{8} a b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]$$

Result (type 5, 79 leaves):

$$\frac{1}{12 x (a + b x^4)^{3/4}} \left(-12 a^2 - 9 a b x^4 + 3 b^2 x^8 + 5 a b x^4 \left(1 + \frac{b x^4}{a} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1064: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^6} dx$$

Optimal (type 3, 92 leaves, 6 steps):

$$-\frac{b (a + b x^4)^{1/4}}{x} - \frac{(a + b x^4)^{5/4}}{5 x^5} - \frac{1}{2} b^{5/4} \operatorname{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right] + \frac{1}{2} b^{5/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]$$

Result (type 5, 81 leaves):

$$\frac{1}{15 x^5 (a + b x^4)^{3/4}} \left(-3 (a^2 + 7 a b x^4 + 6 b^2 x^8) + 5 b^2 x^8 \left(1 + \frac{b x^4}{a} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1069: Result unnecessarily involves higher level functions.

$$\int x^{12} (a + b x^4)^{5/4} dx$$

Optimal (type 4, 171 leaves, 9 steps):

$$\frac{5 a^4 x (a + b x^4)^{1/4}}{672 b^3} - \frac{a^3 x^5 (a + b x^4)^{1/4}}{336 b^2} + \frac{a^2 x^9 (a + b x^4)^{1/4}}{504 b} + \frac{5}{252} a x^{13} (a + b x^4)^{1/4} + \frac{1}{18} x^{13} (a + b x^4)^{5/4} + \frac{5 a^{9/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{672 b^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 112 leaves):

$$\left(15 a^5 x + 9 a^4 b x^5 - 2 a^3 b^2 x^9 + 156 a^2 b^3 x^{13} + 264 a b^4 x^{17} + 112 b^5 x^{21} - 15 a^5 x \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]\right) / \left(2016 b^3 (a + b x^4)^{3/4}\right)$$

Problem 1070: Result unnecessarily involves higher level functions.

$$\int x^8 (a + b x^4)^{5/4} dx$$

Optimal (type 4, 147 leaves, 8 steps):

$$-\frac{5 a^3 x (a + b x^4)^{1/4}}{336 b^2} + \frac{a^2 x^5 (a + b x^4)^{1/4}}{168 b} + \frac{1}{28} a x^9 (a + b x^4)^{1/4} + \frac{1}{14} x^9 (a + b x^4)^{5/4} - \frac{5 a^{7/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{336 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 101 leaves):

$$\left(-5 a^4 x - 3 a^3 b x^5 + 38 a^2 b^2 x^9 + 60 a b^3 x^{13} + 24 b^4 x^{17} + 5 a^4 x \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]\right) / \left(336 b^2 (a + b x^4)^{3/4}\right)$$

Problem 1071: Result unnecessarily involves higher level functions.

$$\int x^4 (a + b x^4)^{5/4} dx$$

Optimal (type 4, 123 leaves, 7 steps):

$$\frac{a^2 x (a + b x^4)^{1/4}}{24 b} + \frac{1}{12} a x^5 (a + b x^4)^{1/4} + \frac{1}{10} x^5 (a + b x^4)^{5/4} + \frac{a^{5/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{24 \sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 90 leaves):

$$\frac{1}{120 b (a + b x^4)^{3/4}} \left(5 a^3 x + 27 a^2 b x^5 + 34 a b^2 x^9 + 12 b^3 x^{13} - 5 a^3 x \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1072: Result unnecessarily involves higher level functions.

$$\int (a + b x^4)^{5/4} dx$$

Optimal (type 4, 97 leaves, 6 steps):

$$\frac{5}{12} a x (a + b x^4)^{1/4} + \frac{1}{6} x (a + b x^4)^{5/4} - \frac{5 a^{3/2} \sqrt{b} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 (a + b x^4)^{3/4}}$$

Result (type 5, 76 leaves):

$$\frac{1}{12 (a + b x^4)^{3/4}} \left(7 a^2 x + 9 a b x^5 + 2 b^2 x^9 + 5 a^2 x \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1073: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^4} dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$\frac{5}{6} b x (a + b x^4)^{1/4} - \frac{(a + b x^4)^{5/4}}{3 x^3} - \frac{5 \sqrt{a} b^{3/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{6 (a + b x^4)^{3/4}}$$

Result (type 5, 80 leaves):

$$\left(-\frac{a}{3 x^3} + \frac{b x}{2}\right) (a + b x^4)^{1/4} + \frac{5 a b x \left(\frac{a + b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{6 (a + b x^4)^{3/4}}$$

Problem 1074: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^4)^{5/4}}{x^8} dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$-\frac{5 b (a+b x^4)^{1/4}}{21 x^3} - \frac{(a+b x^4)^{5/4}}{7 x^7} - \frac{5 b^{5/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{21 \sqrt{a} (a+b x^4)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{21 x^7 (a+b x^4)^{3/4}} \left(-3 a^2 - 11 a b x^4 - 8 b^2 x^8 + 5 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1075: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x^4)^{5/4}}{x^{12}} dx$$

Optimal (type 4, 125 leaves, 7 steps):

$$-\frac{5 b (a+b x^4)^{1/4}}{77 x^7} - \frac{5 b^2 (a+b x^4)^{1/4}}{231 a x^3} - \frac{(a+b x^4)^{5/4}}{11 x^{11}} + \frac{10 b^{7/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{231 a^{3/2} (a+b x^4)^{3/4}}$$

Result (type 5, 94 leaves):

$$\left(-21 a^3 - 57 a^2 b x^4 - 41 a b^2 x^8 - 5 b^3 x^{12} - 10 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right] \right) / \left(231 a x^{11} (a+b x^4)^{3/4} \right)$$

Problem 1076: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x^4)^{5/4}}{x^{16}} dx$$

Optimal (type 4, 149 leaves, 8 steps):

$$-\frac{b (a+b x^4)^{1/4}}{33 x^{11}} - \frac{b^2 (a+b x^4)^{1/4}}{231 a x^7} + \frac{2 b^3 (a+b x^4)^{1/4}}{231 a^2 x^3} - \frac{(a+b x^4)^{5/4}}{15 x^{15}} - \frac{4 b^{9/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{231 a^{5/2} (a+b x^4)^{3/4}}$$

Result (type 5, 105 leaves):

$$\left(-77 a^4 - 189 a^3 b x^4 - 117 a^2 b^2 x^8 + 5 a b^3 x^{12} + 10 b^4 x^{16} + 20 b^4 x^{16} \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right) / \left(1155 a^2 x^{15} (a + b x^4)^{3/4} \right)$$

Problem 1083: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x^4)^{1/4}} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{\text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{2 a^{1/4}} - \frac{\text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{2 a^{1/4}}$$

Result (type 5, 46 leaves):

$$\frac{\left(1 + \frac{a}{b x^4} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4} \right]}{(a + b x^4)^{1/4}}$$

Problem 1084: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (a + b x^4)^{1/4}} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{(a + b x^4)^{3/4}}{4 a x^4} - \frac{b \text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{8 a^{5/4}} + \frac{b \text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{8 a^{5/4}}$$

Result (type 5, 69 leaves):

$$\frac{-a - b x^4 + b \left(1 + \frac{a}{b x^4} \right)^{1/4} x^4 \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4} \right]}{4 a x^4 (a + b x^4)^{1/4}}$$

Problem 1085: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9 (a + b x^4)^{1/4}} dx$$

Optimal (type 3, 104 leaves, 7 steps):

$$-\frac{(a + b x^4)^{3/4}}{8 a x^8} + \frac{5 b (a + b x^4)^{3/4}}{32 a^2 x^4} + \frac{5 b^2 \text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{64 a^{9/4}} - \frac{5 b^2 \text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{64 a^{9/4}}$$

Result (type 5, 82 leaves):

$$\frac{\left(-4 a^2 + a b x^4 + 5 b^2 x^8 - 5 b^2 \left(1 + \frac{a}{b x^4}\right)^{1/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4}\right]\right)}{\left(32 a^2 x^8 (a + b x^4)^{1/4}\right)}$$

Problem 1086: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\begin{aligned} & -\frac{8 a^3 x^2}{39 b^3 (a + b x^4)^{1/4}} + \frac{4 a^2 x^2 (a + b x^4)^{3/4}}{39 b^3} - \frac{10 a x^6 (a + b x^4)^{3/4}}{117 b^2} + \\ & \frac{x^{10} (a + b x^4)^{3/4}}{13 b} + \frac{8 a^{7/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{39 b^{7/2} (a + b x^4)^{1/4}} \end{aligned}$$

Result (type 5, 91 leaves):

$$\frac{\left(x^2 \left(12 a^3 + 2 a^2 b x^4 - a b^2 x^8 + 9 b^3 x^{12} - 12 a^3 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)\right)}{\left(117 b^3 (a + b x^4)^{1/4}\right)}$$

Problem 1087: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\begin{aligned} & \frac{4 a^2 x^2}{15 b^2 (a + b x^4)^{1/4}} - \frac{2 a x^2 (a + b x^4)^{3/4}}{15 b^2} + \\ & \frac{x^6 (a + b x^4)^{3/4}}{9 b} - \frac{4 a^{5/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{15 b^{5/2} (a + b x^4)^{1/4}} \end{aligned}$$

Result (type 5, 80 leaves):

$$\frac{1}{45 b^2 (a + b x^4)^{1/4}} x^2 \left(-6 a^2 - a b x^4 + 5 b^2 x^8 + 6 a^2 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)$$

Problem 1088: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{2 a x^2}{5 b (a + b x^4)^{1/4}} + \frac{x^2 (a + b x^4)^{3/4}}{5 b} + \frac{2 a^{3/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{5 b^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(a + b x^4 - a \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{5 b (a + b x^4)^{1/4}}$$

Problem 1089: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$\frac{x^2}{(a + b x^4)^{1/4}} - \frac{\sqrt{a} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{b} (a + b x^4)^{1/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^2 \left(\frac{a + b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{2 (a + b x^4)^{1/4}}$$

Problem 1090: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$\frac{b x^2}{2 a (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{2 a x^2} - \frac{\sqrt{b} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 \sqrt{a} (a + b x^4)^{1/4}}$$

Result (type 5, 69 leaves):

$$\frac{-2 (a + b x^4) + b x^4 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{4 a x^2 (a + b x^4)^{1/4}}$$

Problem 1091: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$-\frac{b^2 x^2}{4 a^2 (a+b x^4)^{1/4}} - \frac{(a+b x^4)^{3/4}}{6 a x^6} + \frac{b (a+b x^4)^{3/4}}{4 a^2 x^2} + \frac{b^{3/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 a^{3/2} (a+b x^4)^{1/4}}$$

Result (type 5, 83 leaves):

$$\left(-4 a^2 + 2 a b x^4 + 6 b^2 x^8 - 3 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / \left(24 a^2 x^6 (a+b x^4)^{1/4}\right)$$

Problem 1092: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} (a+b x^4)^{1/4}} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\frac{7 b^3 x^2}{40 a^3 (a+b x^4)^{1/4}} - \frac{(a+b x^4)^{3/4}}{10 a x^{10}} + \frac{7 b (a+b x^4)^{3/4}}{60 a^2 x^6} - \frac{7 b^2 (a+b x^4)^{3/4}}{40 a^3 x^2} - \frac{7 b^{5/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 a^{5/2} (a+b x^4)^{1/4}}$$

Result (type 5, 94 leaves):

$$\left(-24 a^3 + 4 a^2 b x^4 - 14 a b^2 x^8 - 42 b^3 x^{12} + 21 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / \left(240 a^3 x^{10} (a+b x^4)^{1/4}\right)$$

Problem 1101: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(a+b x^4)^{1/4}} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{7 a^2 x^3}{40 b^2 (a+b x^4)^{1/4}} - \frac{7 a x^3 (a+b x^4)^{3/4}}{60 b^2} + \frac{x^7 (a+b x^4)^{3/4}}{10 b} + \frac{7 a^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 b^{5/2} (a+b x^4)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{60 b^2 (a+b x^4)^{1/4}} x^3 \left(-7 a^2 - a b x^4 + 6 b^2 x^8 + 7 a^2 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)$$

Problem 1102: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{a x^3}{4 b (a + b x^4)^{1/4}} + \frac{x^3 (a + b x^4)^{3/4}}{6 b} - \frac{a^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 b^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^3 \left(a + b x^4 - a \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)}{6 b (a + b x^4)^{1/4}}$$

Problem 1103: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a + b x^4)^{1/4}} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{x^3}{2 (a + b x^4)^{1/4}} + \frac{\sqrt{a} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 \sqrt{b} (a + b x^4)^{1/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^3 \left(\frac{a + b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]}{3 (a + b x^4)^{1/4}}$$

Problem 1104: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 75 leaves, 5 steps):

$$-\frac{1}{x (a + b x^4)^{1/4}} + \frac{\sqrt{b} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{a} (a + b x^4)^{1/4}}$$

Result (type 5, 70 leaves):

$$\frac{-3 (a + b x^4) + 2 b x^4 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]}{3 a x (a + b x^4)^{1/4}}$$

Problem 1105: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^6 (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$\frac{2 b}{5 a x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{5 a x^5} - \frac{2 b^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{5 a^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 83 leaves):

$$\left(-3 a^2 + 3 a b x^4 + 6 b^2 x^8 - 4 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right) / (15 a^2 x^5 (a + b x^4)^{1/4})$$

Problem 1106: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{10} (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{4 b^2}{15 a^2 x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{9 a x^9} + \frac{2 b (a + b x^4)^{3/4}}{15 a^2 x^5} + \frac{4 b^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{15 a^{5/2} (a + b x^4)^{1/4}}$$

Result (type 5, 93 leaves):

$$\left(-5 a^3 + a^2 b x^4 - 6 a b^2 x^8 - 12 b^3 x^{12} + 8 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right) / (45 a^3 x^9 (a + b x^4)^{1/4})$$

Problem 1107: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{14} (a + b x^4)^{1/4}} dx$$

Optimal (type 4, 153 leaves, 8 steps):

$$\frac{8 b^3}{39 a^3 x (a + b x^4)^{1/4}} - \frac{(a + b x^4)^{3/4}}{13 a x^{13}} + \frac{10 b (a + b x^4)^{3/4}}{117 a^2 x^9} - \frac{4 b^2 (a + b x^4)^{3/4}}{39 a^3 x^5} - \frac{8 b^{7/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{39 a^{7/2} (a + b x^4)^{1/4}}$$

Result (type 5, 104 leaves):

$$\left(-9 a^4 + a^3 b x^4 - 2 a^2 b^2 x^8 + 12 a b^3 x^{12} + 24 b^4 x^{16} - 16 b^4 x^{16} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right) / \left(117 a^4 x^{13} (a + b x^4)^{1/4} \right)$$

Problem 1113: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x^4)^{3/4}} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$-\frac{\text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{2 a^{3/4}} - \frac{\text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{2 a^{3/4}}$$

Result (type 5, 48 leaves):

$$-\frac{\left(1 + \frac{a}{b x^4} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4} \right]}{3 (a + b x^4)^{3/4}}$$

Problem 1114: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (a + b x^4)^{3/4}} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{(a + b x^4)^{1/4}}{4 a x^4} + \frac{3 b \text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{8 a^{7/4}} + \frac{3 b \text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{8 a^{7/4}}$$

Result (type 5, 69 leaves):

$$\frac{-a - b x^4 + b \left(1 + \frac{a}{b x^4} \right)^{3/4} x^4 \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4} \right]}{4 a x^4 (a + b x^4)^{3/4}}$$

Problem 1115: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9 (a + b x^4)^{3/4}} dx$$

Optimal (type 3, 104 leaves, 7 steps):

$$-\frac{(a + b x^4)^{1/4}}{8 a x^8} + \frac{7 b (a + b x^4)^{1/4}}{32 a^2 x^4} - \frac{21 b^2 \text{ArcTan} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{64 a^{11/4}} - \frac{21 b^2 \text{ArcTanh} \left[\frac{(a + b x^4)^{1/4}}{a^{1/4}} \right]}{64 a^{11/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-4 a^2 + 3 a b x^4 + 7 b^2 x^8 - 7 b^2 \left(1 + \frac{a}{b x^4}\right)^{3/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{a}{b x^4}\right]\right)}{\left(32 a^2 x^8 (a + b x^4)^{3/4}\right)}$$

Problem 1116: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{20 a^2 x^2 (a + b x^4)^{1/4}}{77 b^3} - \frac{10 a x^6 (a + b x^4)^{1/4}}{77 b^2} + \frac{x^{10} (a + b x^4)^{1/4}}{11 b} - \frac{40 a^{7/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{77 b^{7/2} (a + b x^4)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{1}{77 b^3 (a + b x^4)^{3/4}} x^2 \left(20 a^3 + 10 a^2 b x^4 - 3 a b^2 x^8 + 7 b^3 x^{12} - 20 a^3 \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)$$

Problem 1117: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{2 a x^2 (a + b x^4)^{1/4}}{7 b^2} + \frac{x^6 (a + b x^4)^{1/4}}{7 b} + \frac{4 a^{5/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{7 b^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{7 b^2 (a + b x^4)^{3/4}} x^2 \left(-2 a^2 - a b x^4 + b^2 x^8 + 2 a^2 \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)$$

Problem 1118: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{x^2 (a + b x^4)^{1/4}}{3 b} - \frac{2 a^{3/2} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(a + b x^4 - a \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{3 b (a + b x^4)^{3/4}}$$

Problem 1119: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{\sqrt{a} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^2 \left(\frac{a + b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{2 (a + b x^4)^{3/4}}$$

Problem 1120: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a + b x^4)^{3/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{(a + b x^4)^{1/4}}{2 a x^2} - \frac{\sqrt{b} \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 \sqrt{a} (a + b x^4)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-2 (a + b x^4) - b x^4 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{4 a x^2 (a + b x^4)^{3/4}}$$

Problem 1121: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 (a + b x^4)^{3/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{(a+bx^4)^{1/4}}{6ax^6} + \frac{5b(a+bx^4)^{1/4}}{12a^2x^2} + \frac{5b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{12a^{3/2}(a+bx^4)^{3/4}}$$

Result (type 5, 83 leaves):

$$\left(-4a^2 + 6abx^4 + 10b^2x^8 + 5b^2x^8\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{bx^4}{a}\right]\right) / \left(24a^2x^6(a+bx^4)^{3/4}\right)$$

Problem 1122: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11}(a+bx^4)^{3/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$-\frac{(a+bx^4)^{1/4}}{10ax^{10}} + \frac{3b(a+bx^4)^{1/4}}{20a^2x^6} - \frac{3b^2(a+bx^4)^{1/4}}{8a^3x^2} - \frac{3b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{8a^{5/2}(a+bx^4)^{3/4}}$$

Result (type 5, 94 leaves):

$$\left(-8a^3 + 4a^2bx^4 - 18ab^2x^8 - 30b^3x^{12} - 15b^3x^{12}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{bx^4}{a}\right]\right) / \left(80a^3x^{10}(a+bx^4)^{3/4}\right)$$

Problem 1123: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}} dx$$

Optimal (type 3, 106 leaves, 6 steps):

$$-\frac{7ax^3(a+bx^4)^{1/4}}{32b^2} + \frac{x^7(a+bx^4)^{1/4}}{8b} - \frac{21a^2 \text{ArcTan}\left[\frac{b^{1/4}x}{(a+bx^4)^{1/4}}\right]}{64b^{11/4}} + \frac{21a^2 \text{ArcTanh}\left[\frac{b^{1/4}x}{(a+bx^4)^{1/4}}\right]}{64b^{11/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{32b^2(a+bx^4)^{3/4}} x^3 \left(-7a^2 - 3abx^4 + 4b^2x^8 + 7a^2\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^4}{a}\right]\right)$$

Problem 1124: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a + b x^4)^{3/4}} dx$$

Optimal (type 3, 80 leaves, 5 steps):

$$\frac{x^3 (a + b x^4)^{1/4}}{4 b} + \frac{3 a \operatorname{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{8 b^{7/4}} - \frac{3 a \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{8 b^{7/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^3 \left(a + b x^4 - a \left(1 + \frac{b x^4}{a} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)}{4 b (a + b x^4)^{3/4}}$$

Problem 1125: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a + b x^4)^{3/4}} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{2 b^{3/4}} + \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} x}{(a + b x^4)^{1/4}}\right]}{2 b^{3/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^3 \left(\frac{a + b x^4}{a} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]}{3 (a + b x^4)^{3/4}}$$

Problem 1130: Result unnecessarily involves higher level functions.

$$\int \frac{x^{12}}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{3 a^2 x (a + b x^4)^{1/4}}{8 b^3} - \frac{3 a x^5 (a + b x^4)^{1/4}}{20 b^2} + \frac{x^9 (a + b x^4)^{1/4}}{10 b} + \frac{3 a^{5/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{8 b^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 90 leaves):

$$\frac{1}{40 b^3 (a + b x^4)^{3/4}} \left(15 a^3 x + 9 a^2 b x^5 - 2 a b^2 x^9 + 4 b^3 x^{13} - 15 a^3 x \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)$$

Problem 1131: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{5 a x (a + b x^4)^{1/4}}{12 b^2} + \frac{x^5 (a + b x^4)^{1/4}}{6 b} - \frac{5 a^{3/2} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{12 b^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{12 b^2 (a + b x^4)^{3/4}} \left(-5 a^2 x - 3 a b x^5 + 2 b^2 x^9 + 5 a^2 x \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)$$

Problem 1132: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{x (a + b x^4)^{1/4}}{2 b} + \frac{\sqrt{a} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{2 \sqrt{b} (a + b x^4)^{3/4}}$$

Result (type 5, 62 leaves):

$$\frac{x \left(a + b x^4 - a \left(1 + \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a} \right] \right)}{2 b (a + b x^4)^{3/4}}$$

Problem 1133: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^4)^{3/4}} dx$$

Optimal (type 4, 61 leaves, 4 steps):

$$-\frac{\sqrt{b} \left(1 + \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{\sqrt{a} (a + b x^4)^{3/4}}$$

Result (type 5, 47 leaves):

$$\frac{x \left(\frac{a + b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{(a + b x^4)^{3/4}}$$

Problem 1134: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a + b x^4)^{3/4}} dx$$

Optimal (type 4, 85 leaves, 5 steps):

$$-\frac{(a + b x^4)^{1/4}}{3 a x^3} + \frac{2 b^{3/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 a^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a - b x^4 - 2 b x^4 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{3 a x^3 (a + b x^4)^{3/4}}$$

Problem 1135: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8 (a + b x^4)^{3/4}} dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$-\frac{(a + b x^4)^{1/4}}{7 a x^7} + \frac{2 b (a + b x^4)^{1/4}}{7 a^2 x^3} - \frac{4 b^{5/2} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{7 a^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 82 leaves):

$$\frac{\left(-a^2 + a b x^4 + 2 b^2 x^8 + 4 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]\right)}{\left(7 a^2 x^7 (a + b x^4)^{3/4}\right)}$$

Problem 1136: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{12} (a + b x^4)^{3/4}} dx$$

Optimal (type 4, 131 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{(a+bx^4)^{1/4}}{11ax^{11}} + \frac{10b(a+bx^4)^{1/4}}{77a^2x^7} - \frac{20b^2(a+bx^4)^{1/4}}{77a^3x^3} + \\
 & \frac{40b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCot}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{77a^{7/2}(a+bx^4)^{3/4}}
 \end{aligned}$$

Result (type 5, 94 leaves):

$$\begin{aligned}
 & \left(-7a^3 + 3a^2bx^4 - 10ab^2x^8 - 20b^3x^{12} - \right. \\
 & \left. 40b^3x^{12}\left(1+\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{bx^4}{a}\right]\right) / \left(77a^3x^{11}(a+bx^4)^{3/4}\right)
 \end{aligned}$$

Problem 1142: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x(a+bx^4)^{5/4}} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{1}{a(a+bx^4)^{1/4}} + \frac{\operatorname{ArcTan}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{5/4}} - \frac{\operatorname{ArcTanh}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{5/4}}$$

Result (type 5, 52 leaves):

$$\frac{1 - \left(1 + \frac{a}{bx^4}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{bx^4}\right]}{a(a+bx^4)^{1/4}}$$

Problem 1143: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5(a+bx^4)^{5/4}} dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{5b}{4a^2(a+bx^4)^{1/4}} - \frac{1}{4ax^4(a+bx^4)^{1/4}} - \frac{5b \operatorname{ArcTan}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]}{8a^{9/4}} + \frac{5b \operatorname{ArcTanh}\left[\frac{(a+bx^4)^{1/4}}{a^{1/4}}\right]}{8a^{9/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a - 5bx^4 + 5b\left(1+\frac{a}{bx^4}\right)^{1/4}x^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{bx^4}\right]}{4a^2x^4(a+bx^4)^{1/4}}$$

Problem 1144: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9 (a + b x^4)^{5/4}} dx$$

Optimal (type 3, 125 leaves, 8 steps):

$$\frac{45 b^2}{32 a^3 (a + b x^4)^{1/4}} - \frac{1}{8 a x^8 (a + b x^4)^{1/4}} + \frac{9 b}{32 a^2 x^4 (a + b x^4)^{1/4}} + \frac{45 b^2 \operatorname{ArcTan}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{13/4}} - \frac{45 b^2 \operatorname{ArcTanh}\left[\frac{(a + b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{13/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-4 a^2 + 9 a b x^4 + 45 b^2 x^8 - 45 b^2 \left(1 + \frac{a}{b x^4}\right)^{1/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^4}\right]\right)}{\left(32 a^3 x^8 (a + b x^4)^{1/4}\right)}$$

Problem 1145: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{(a + b x^4)^{5/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{4 a^2 x^2}{3 b^3 (a + b x^4)^{1/4}} - \frac{2 a x^6}{9 b^2 (a + b x^4)^{1/4}} + \frac{x^{10}}{9 b (a + b x^4)^{1/4}} - \frac{8 a^{5/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 b^{7/2} (a + b x^4)^{1/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{9 b^3 (a + b x^4)^{1/4}} x^2 \left(-12 a^2 - 2 a b x^4 + b^2 x^8 + 12 a^2 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)$$

Problem 1146: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a + b x^4)^{5/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{6 a x^2}{5 b^2 (a + b x^4)^{1/4}} + \frac{x^6}{5 b (a + b x^4)^{1/4}} + \frac{12 a^{3/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{5 b^{5/2} (a + b x^4)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^2 \left(6 a + b x^4 - 6 a \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{5 b^2 (a + b x^4)^{1/4}}$$

Problem 1147: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{(a + b x^4)^{5/4}} dx$$

Optimal (type 4, 77 leaves, 4 steps):

$$\frac{x^2}{b (a + b x^4)^{1/4}} - \frac{2 \sqrt{a} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{b^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 54 leaves):

$$\frac{x^2 \left(-1 + \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{b (a + b x^4)^{1/4}}$$

Problem 1148: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a + b x^4)^{5/4}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{\left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{a} \sqrt{b} (a + b x^4)^{1/4}}$$

Result (type 5, 57 leaves):

$$-\frac{x^2 \left(-2 + \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right)}{2 a (a + b x^4)^{1/4}}$$

Problem 1149: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a + b x^4)^{5/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{1}{2 a x^2 (a + b x^4)^{1/4}} - \frac{3 \sqrt{b} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 a^{3/2} (a + b x^4)^{1/4}}$$

Result (type 5, 71 leaves):

$$\left(-2 (a + 3 b x^4) + 3 b x^4 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / \left(4 a^2 x^2 (a + b x^4)^{1/4}\right)$$

Problem 1150: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 (a + b x^4)^{5/4}} dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{1}{6 a x^6 (a + b x^4)^{1/4}} + \frac{7 b}{12 a^2 x^2 (a + b x^4)^{1/4}} + \frac{7 b^{3/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 a^{5/2} (a + b x^4)^{1/4}}$$

Result (type 5, 83 leaves):

$$\left(-4 a^2 + 14 a b x^4 + 42 b^2 x^8 - 21 b^2 x^8 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / \left(24 a^3 x^6 (a + b x^4)^{1/4}\right)$$

Problem 1151: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} (a + b x^4)^{5/4}} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$-\frac{1}{10 a x^{10} (a + b x^4)^{1/4}} + \frac{11 b}{60 a^2 x^6 (a + b x^4)^{1/4}} - \frac{77 b^2}{120 a^3 x^2 (a + b x^4)^{1/4}} - \frac{77 b^{5/2} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 a^{7/2} (a + b x^4)^{1/4}}$$

Result (type 5, 94 leaves):

$$\left(-24 a^3 + 44 a^2 b x^4 - 154 a b^2 x^8 - 462 b^3 x^{12} + 231 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]\right) / \left(240 a^4 x^{10} (a + b x^4)^{1/4}\right)$$

Problem 1160: Result unnecessarily involves higher level functions.

$$\int \frac{x^{14}}{(a+b x^4)^{5/4}} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{77 a^2 x^3}{120 b^3 (a+b x^4)^{1/4}} - \frac{11 a x^7}{60 b^2 (a+b x^4)^{1/4}} + \frac{x^{11}}{10 b (a+b x^4)^{1/4}} + \frac{77 a^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 b^{7/2} (a+b x^4)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{60 b^3 (a+b x^4)^{1/4}} x^3 \left(-77 a^2 - 11 a b x^4 + 6 b^2 x^8 + 77 a^2 \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1161: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(a+b x^4)^{5/4}} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{7 a x^3}{12 b^2 (a+b x^4)^{1/4}} + \frac{x^7}{6 b (a+b x^4)^{1/4}} - \frac{7 a^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 b^{5/2} (a+b x^4)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^3 \left(7 a + b x^4 - 7 a \left(1 + \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right] \right)}{6 b^2 (a+b x^4)^{1/4}}$$

Problem 1162: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a+b x^4)^{5/4}} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{x^3}{2 b (a+b x^4)^{1/4}} + \frac{3 \sqrt{a} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 b^{3/2} (a+b x^4)^{1/4}}$$

Result (type 5, 54 leaves):

$$\frac{x^3 \left(-1 + \left(1 + \frac{bx^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^4}{a} \right] \right)}{b (a + bx^4)^{1/4}}$$

Problem 1163: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a + bx^4)^{5/4}} dx$$

Optimal (type 4, 59 leaves, 4 steps):

$$-\frac{\left(1 + \frac{a}{bx^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{\sqrt{a} \sqrt{b} (a + bx^4)^{1/4}}$$

Result (type 5, 58 leaves):

$$-\frac{x^3 \left(-3 + 2 \left(1 + \frac{bx^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^4}{a} \right] \right)}{3 a (a + bx^4)^{1/4}}$$

Problem 1164: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a + bx^4)^{5/4}} dx$$

Optimal (type 4, 79 leaves, 5 steps):

$$-\frac{1}{ax (a + bx^4)^{1/4}} + \frac{2 \sqrt{b} \left(1 + \frac{a}{bx^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{a^{3/2} (a + bx^4)^{1/4}}$$

Result (type 5, 71 leaves):

$$\left(-3 (a + 2bx^4) + 4bx^4 \left(1 + \frac{bx^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^4}{a} \right] \right) / \left(3a^2 x (a + bx^4)^{1/4} \right)$$

Problem 1165: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^6 (a + bx^4)^{5/4}} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{1}{5ax^5 (a + bx^4)^{1/4}} + \frac{6b}{5a^2 x (a + bx^4)^{1/4}} - \frac{12b^{3/2} \left(1 + \frac{a}{bx^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{5a^{5/2} (a + bx^4)^{1/4}}$$

Result (type 5, 83 leaves):

$$\left(-a^2 + 6 a b x^4 + 12 b^2 x^8 - 8 b^2 x^8 \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right) / \left(5 a^3 x^5 (a + b x^4)^{1/4} \right)$$

Problem 1166: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{10} (a + b x^4)^{5/4}} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$-\frac{1}{9 a x^9 (a + b x^4)^{1/4}} + \frac{2 b}{9 a^2 x^5 (a + b x^4)^{1/4}} - \frac{4 b^2}{3 a^3 x (a + b x^4)^{1/4}} + \frac{8 b^{5/2} \left(1 + \frac{a}{b x^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{3 a^{7/2} (a + b x^4)^{1/4}}$$

Result (type 5, 94 leaves):

$$\left(-a^3 + 2 a^2 b x^4 - 12 a b^2 x^8 - 24 b^3 x^{12} + 16 b^3 x^{12} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right) / \left(9 a^4 x^9 (a + b x^4)^{1/4} \right)$$

Problem 1167: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{14} (a + b x^4)^{5/4}} dx$$

Optimal (type 4, 153 leaves, 8 steps):

$$-\frac{1}{13 a x^{13} (a + b x^4)^{1/4}} + \frac{14 b}{117 a^2 x^9 (a + b x^4)^{1/4}} - \frac{28 b^2}{117 a^3 x^5 (a + b x^4)^{1/4}} + \frac{56 b^3}{39 a^4 x (a + b x^4)^{1/4}} - \frac{112 b^{7/2} \left(1 + \frac{a}{b x^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCot} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{39 a^{9/2} (a + b x^4)^{1/4}}$$

Result (type 5, 105 leaves):

$$\left(-9 a^4 + 14 a^3 b x^4 - 28 a^2 b^2 x^8 + 168 a b^3 x^{12} + 336 b^4 x^{16} - 224 b^4 x^{16} \left(1 + \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a} \right] \right) / \left(117 a^5 x^{13} (a + b x^4)^{1/4} \right)$$

Problem 1168: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^4)^{7/4}} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{x}{3 a (a + b x^4)^{3/4}} - \frac{2 \sqrt{b} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 a^{3/2} (a + b x^4)^{3/4}}$$

Result (type 5, 56 leaves):

$$\frac{x + 2 x \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{3 a (a + b x^4)^{3/4}}$$

Problem 1170: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^4)^{11/4}} dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{x}{7 a (a + b x^4)^{7/4}} + \frac{2 x}{7 a^2 (a + b x^4)^{3/4}} - \frac{4 \sqrt{b} \left(1 + \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{7 a^{5/2} (a + b x^4)^{3/4}}$$

Result (type 5, 72 leaves):

$$\frac{1}{7 a^2 (a + b x^4)^{7/4}} \left(3 a x + 2 b x^5 + 4 x (a + b x^4) \left(1 + \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right] \right)$$

Problem 1178: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x} dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$(a - b x^4)^{1/4} - \frac{1}{2} a^{1/4} \operatorname{ArcTan}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right] - \frac{1}{2} a^{1/4} \operatorname{ArcTanh}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 63 leaves):

$$(a - b x^4)^{1/4} - \frac{a \left(1 - \frac{a}{b x^4}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{b x^4}\right]}{3 (a - b x^4)^{3/4}}$$

Problem 1179: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^5} dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{(a-bx^4)^{1/4}}{4x^4} + \frac{b \operatorname{ArcTan}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{8a^{3/4}} + \frac{b \operatorname{ArcTanh}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{8a^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{-3a + 3bx^4 + b\left(1 - \frac{a}{bx^4}\right)^{3/4} x^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{bx^4}\right]}{12x^4 (a-bx^4)^{3/4}}$$

Problem 1180: Result unnecessarily involves higher level functions.

$$\int \frac{(a-bx^4)^{1/4}}{x^9} dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{(a-bx^4)^{1/4}}{8x^8} + \frac{b(a-bx^4)^{1/4}}{32ax^4} + \frac{3b^2 \operatorname{ArcTan}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{64a^{7/4}} + \frac{3b^2 \operatorname{ArcTanh}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{64a^{7/4}}$$

Result (type 5, 83 leaves):

$$\frac{\left(-4a^2 + 5abx^4 - b^2x^8 + b^2\left(1 - \frac{a}{bx^4}\right)^{3/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{bx^4}\right]\right)}{\left(32ax^8 (a-bx^4)^{3/4}\right)}$$

Problem 1181: Result unnecessarily involves higher level functions.

$$\int x^9 (a-bx^4)^{1/4} dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$-\frac{2a^2x^2(a-bx^4)^{1/4}}{77b^2} - \frac{ax^6(a-bx^4)^{1/4}}{77b} + \frac{\frac{1}{11}x^{10}(a-bx^4)^{1/4} + \frac{4a^{7/2}\left(1 - \frac{bx^4}{a}\right)^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{77b^{5/2}(a-bx^4)^{3/4}}}{}$$

Result (type 5, 91 leaves):

$$\frac{1}{77b^2(a-bx^4)^{3/4}} x^2 \left(-2a^3 + a^2bx^4 + 8ab^2x^8 - 7b^3x^{12} + 2a^3\left(1 - \frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^4}{a}\right]\right)$$

Problem 1182: Result unnecessarily involves higher level functions.

$$\int x^5 (a-bx^4)^{1/4} dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$-\frac{a x^2 (a - b x^4)^{1/4}}{21 b} + \frac{1}{7} x^6 (a - b x^4)^{1/4} + \frac{2 a^{5/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{21 b^{3/2} (a - b x^4)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{21 b (a - b x^4)^{3/4}} x^2 \left(-a^2 + 4 a b x^4 - 3 b^2 x^8 + a^2 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]\right)$$

Problem 1183: Result unnecessarily involves higher level functions.

$$\int x (a - b x^4)^{1/4} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{1}{3} x^2 (a - b x^4)^{1/4} + \frac{a^{3/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 \sqrt{b} (a - b x^4)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(2 a - 2 b x^4 + a \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]\right)}{6 (a - b x^4)^{3/4}}$$

Problem 1184: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^3} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{(a - b x^4)^{1/4}}{2 x^2} - \frac{\sqrt{a} \sqrt{b} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 (a - b x^4)^{3/4}}$$

Result (type 5, 68 leaves):

$$\frac{-2 a + 2 b x^4 - b x^4 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]}{4 x^2 (a - b x^4)^{3/4}}$$

Problem 1185: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^7} dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$-\frac{(a - b x^4)^{1/4}}{6 x^6} + \frac{b (a - b x^4)^{1/4}}{12 a x^2} - \frac{b^{3/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 \sqrt{a} (a - b x^4)^{3/4}}$$

Result (type 5, 84 leaves):

$$\left(-4 a^2 + 6 a b x^4 - 2 b^2 x^8 - b^2 x^8 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]\right) / (24 a x^6 (a - b x^4)^{3/4})$$

Problem 1186: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^{11}} dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$-\frac{(a - b x^4)^{1/4}}{10 x^{10}} + \frac{b (a - b x^4)^{1/4}}{60 a x^6} + \frac{b^2 (a - b x^4)^{1/4}}{24 a^2 x^2} - \frac{b^{5/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{24 a^{3/2} (a - b x^4)^{3/4}}$$

Result (type 5, 95 leaves):

$$\left(-24 a^3 + 28 a^2 b x^4 + 6 a b^2 x^8 - 10 b^3 x^{12} - 5 b^3 x^{12} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]\right) / (240 a^2 x^{10} (a - b x^4)^{3/4})$$

Problem 1187: Result unnecessarily involves higher level functions.

$$\int x^6 (a - b x^4)^{1/4} dx$$

Optimal (type 3, 263 leaves, 12 steps):

$$-\frac{a x^3 (a - b x^4)^{1/4}}{32 b} + \frac{1}{8} x^7 (a - b x^4)^{1/4} - \frac{3 a^2 \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{64 \sqrt{2} b^{7/4}} + \frac{3 a^2 \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{64 \sqrt{2} b^{7/4}} + \frac{3 a^2 \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{128 \sqrt{2} b^{7/4}} - \frac{3 a^2 \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{128 \sqrt{2} b^{7/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{32 b (a - b x^4)^{3/4}} x^3 \left(-a^2 + 5 a b x^4 - 4 b^2 x^8 + a^2 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]\right)$$

Problem 1188: Result unnecessarily involves higher level functions.

$$\int x^2 (a - b x^4)^{1/4} dx$$

Optimal (type 3, 232 leaves, 11 steps):

$$\frac{1}{4} x^3 (a - b x^4)^{1/4} - \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{8 \sqrt{2} b^{3/4}} + \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{8 \sqrt{2} b^{3/4}} +$$

$$\frac{a \operatorname{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{16 \sqrt{2} b^{3/4}} - \frac{a \operatorname{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{16 \sqrt{2} b^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^3 \left(3 a - 3 b x^4 + a \left(1 - \frac{b x^4}{a}\right)^{3/4}\right) \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]}{12 (a - b x^4)^{3/4}}$$

Problem 1189: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^2} dx$$

Optimal (type 3, 226 leaves, 11 steps):

$$-\frac{(a - b x^4)^{1/4}}{x} + \frac{b^{1/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{2 \sqrt{2}} - \frac{b^{1/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{2 \sqrt{2}} -$$

$$\frac{b^{1/4} \operatorname{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{4 \sqrt{2}} + \frac{b^{1/4} \operatorname{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{4 \sqrt{2}}$$

Result (type 5, 68 leaves):

$$\frac{-3 a + 3 b x^4 - b x^4 \left(1 - \frac{b x^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]}{3 x (a - b x^4)^{3/4}}$$

Problem 1194: Result unnecessarily involves higher level functions.

$$\int x^{12} (a - b x^4)^{1/4} dx$$

Optimal (type 4, 156 leaves, 8 steps):

$$-\frac{3 a^3 x (a - b x^4)^{1/4}}{112 b^3} - \frac{3 a^2 x^5 (a - b x^4)^{1/4}}{280 b^2} - \frac{a x^9 (a - b x^4)^{1/4}}{140 b} +$$

$$\frac{1}{14} x^{13} (a - b x^4)^{1/4} - \frac{3 a^{7/2} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{112 b^{5/2} (a - b x^4)^{3/4}}$$

Result (type 5, 102 leaves):

$$\left(-15 a^4 x + 9 a^3 b x^5 + 2 a^2 b^2 x^9 + 44 a b^3 x^{13} - 40 b^4 x^{17} + 15 a^4 x \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a} \right] \right) / \left(560 b^3 (a - b x^4)^{3/4} \right)$$

Problem 1195: Result unnecessarily involves higher level functions.

$$\int x^8 (a - b x^4)^{1/4} dx$$

Optimal (type 4, 131 leaves, 7 steps):

$$-\frac{a^2 x (a - b x^4)^{1/4}}{24 b^2} - \frac{a x^5 (a - b x^4)^{1/4}}{60 b} + \frac{\frac{1}{10} x^9 (a - b x^4)^{1/4} - \frac{a^{5/2} \left(1 - \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCsc} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{24 b^{3/2} (a - b x^4)^{3/4}}$$

Result (type 5, 91 leaves):

$$\left(-5 a^3 x + 3 a^2 b x^5 + 14 a b^2 x^9 - 12 b^3 x^{13} + 5 a^3 x \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a} \right] \right) / \left(120 b^2 (a - b x^4)^{3/4} \right)$$

Problem 1196: Result unnecessarily involves higher level functions.

$$\int x^4 (a - b x^4)^{1/4} dx$$

Optimal (type 4, 106 leaves, 6 steps):

$$-\frac{a x (a - b x^4)^{1/4}}{12 b} + \frac{1}{6} x^5 (a - b x^4)^{1/4} - \frac{a^{3/2} \left(1 - \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCsc} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{12 \sqrt{b} (a - b x^4)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{12 b (a - b x^4)^{3/4}} \left(-a^2 x + 3 a b x^5 - 2 b^2 x^9 + a^2 x \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a} \right] \right)$$

Problem 1197: Result unnecessarily involves higher level functions.

$$\int (a - b x^4)^{1/4} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{1}{2} x (a - b x^4)^{1/4} - \frac{\sqrt{a} \sqrt{b} \left(1 - \frac{a}{b x^4} \right)^{3/4} x^3 \text{EllipticF} \left[\frac{1}{2} \text{ArcCsc} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{2 (a - b x^4)^{3/4}}$$

Result (type 5, 62 leaves):

$$\frac{a x - b x^5 + a x \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]}{2 (a - b x^4)^{3/4}}$$

Problem 1198: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^4} dx$$

Optimal (type 4, 85 leaves, 5 steps):

$$-\frac{(a - b x^4)^{1/4}}{3 x^3} + \frac{b^{3/2} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 \sqrt{a} (a - b x^4)^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{-a + b x^4 - b x^4 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]}{3 x^3 (a - b x^4)^{3/4}}$$

Problem 1199: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^8} dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$-\frac{(a - b x^4)^{1/4}}{7 x^7} + \frac{b (a - b x^4)^{1/4}}{21 a x^3} + \frac{2 b^{5/2} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{21 a^{3/2} (a - b x^4)^{3/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(-3 a^2 + 4 a b x^4 - b^2 x^8 - 2 b^2 x^8 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]\right)}{(21 a x^7 (a - b x^4)^{3/4})}$$

Problem 1200: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x^4)^{1/4}}{x^{12}} dx$$

Optimal (type 4, 133 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{(a-bx^4)^{1/4}}{11x^{11}} + \frac{b(a-bx^4)^{1/4}}{77ax^7} + \frac{2b^2(a-bx^4)^{1/4}}{77a^2x^3} + \\
 & \frac{4b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{77a^{5/2}(a-bx^4)^{3/4}}
 \end{aligned}$$

Result (type 5, 94 leaves):

$$\begin{aligned}
 & \left(-7a^3 + 8a^2bx^4 + a^2bx^8 - 2b^3x^{12} - 4b^3x^{12}\left(1-\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^4}{a}\right]\right) / \\
 & (77a^2x^{11}(a-bx^4)^{3/4})
 \end{aligned}$$

Problem 1201: Result unnecessarily involves higher level functions.

$$\int \frac{(a-bx^4)^{1/4}}{x^{16}} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{(a-bx^4)^{1/4}}{15x^{15}} + \frac{b(a-bx^4)^{1/4}}{165ax^{11}} + \frac{2b^2(a-bx^4)^{1/4}}{231a^2x^7} + \\
 & \frac{4b^3(a-bx^4)^{1/4}}{231a^3x^3} + \frac{8b^{9/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{231a^{7/2}(a-bx^4)^{3/4}}
 \end{aligned}$$

Result (type 5, 106 leaves):

$$\begin{aligned}
 & \left(-77a^4 + 84a^3bx^4 + 3a^2b^2x^8 + 10ab^3x^{12} - 20b^4x^{16} - \right. \\
 & \left. 40b^4x^{16}\left(1-\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^4}{a}\right]\right) / (1155a^3x^{15}(a-bx^4)^{3/4})
 \end{aligned}$$

Problem 1207: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x(a-bx^4)^{1/4}} dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{1/4}} - \frac{\operatorname{ArcTanh}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{1/4}}$$

Result (type 5, 47 leaves):

$$\frac{\left(1-\frac{a}{bx^4}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{a}{bx^4}\right]}{(a-bx^4)^{1/4}}$$

Problem 1208: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (a - b x^4)^{1/4}} dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{(a - b x^4)^{3/4}}{4 a x^4} + \frac{b \operatorname{ArcTan}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}} - \frac{b \operatorname{ArcTanh}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a + b x^4 - b \left(1 - \frac{a}{b x^4}\right)^{1/4} x^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{a}{b x^4}\right]}{4 a x^4 (a - b x^4)^{1/4}}$$

Problem 1209: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9 (a - b x^4)^{1/4}} dx$$

Optimal (type 3, 108 leaves, 7 steps):

$$-\frac{(a - b x^4)^{3/4}}{8 a x^8} - \frac{5 b (a - b x^4)^{3/4}}{32 a^2 x^4} + \frac{5 b^2 \operatorname{ArcTan}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}} - \frac{5 b^2 \operatorname{ArcTanh}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(-4 a^2 - a b x^4 + 5 b^2 x^8 - 5 b^2 \left(1 - \frac{a}{b x^4}\right)^{1/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{a}{b x^4}\right]\right)}{\left(32 a^2 x^8 (a - b x^4)^{1/4}\right)}$$

Problem 1210: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\frac{4 a^2 x^2 (a - b x^4)^{3/4}}{39 b^3} - \frac{10 a x^6 (a - b x^4)^{3/4}}{117 b^2} - \frac{x^{10} (a - b x^4)^{3/4}}{13 b} + \frac{8 a^{7/2} \left(1 - \frac{b x^4}{a}\right)^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{39 b^{7/2} (a - b x^4)^{1/4}}$$

Result (type 5, 91 leaves):

$$\frac{\left(x^2 \left(-12 a^3 + 2 a^2 b x^4 + a b^2 x^8 + 9 b^3 x^{12} + 12 a^3 \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{b x^4}{a} \right] \right) \right)^{1/4}}{(117 b^3 (a - b x^4)^{1/4})}$$

Problem 1211: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{2 a x^2 (a - b x^4)^{3/4}}{15 b^2} - \frac{x^6 (a - b x^4)^{3/4}}{9 b} + \frac{4 a^{5/2} \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{15 b^{5/2} (a - b x^4)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{45 b^2 (a - b x^4)^{1/4}} x^2 \left(-6 a^2 + a b x^4 + 5 b^2 x^8 + 6 a^2 \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{b x^4}{a} \right] \right)$$

Problem 1212: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{x^2 (a - b x^4)^{3/4}}{5 b} + \frac{2 a^{3/2} \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{5 b^{3/2} (a - b x^4)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^2 \left(-a + b x^4 + a \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{b x^4}{a} \right] \right)}{5 b (a - b x^4)^{1/4}}$$

Problem 1213: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{\sqrt{a} \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{EllipticE} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{\sqrt{b} (a - b x^4)^{1/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^2 \left(\frac{a-bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^4}{a}\right]}{2(a-bx^4)^{1/4}}$$

Problem 1214: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{(a-bx^4)^{3/4}}{2ax^2} - \frac{\sqrt{b} \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{2\sqrt{a}(a-bx^4)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-2a + 2bx^4 - bx^4 \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^4}{a}\right]}{4ax^2(a-bx^4)^{1/4}}$$

Problem 1215: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{(a-bx^4)^{3/4}}{6ax^6} - \frac{b(a-bx^4)^{3/4}}{4a^2x^2} - \frac{b^{3/2} \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{4a^{3/2}(a-bx^4)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(-4a^2 - 2abx^4 + 6b^2x^8 - 3b^2x^8 \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^4}{a}\right]\right)}{\left(24a^2x^6(a-bx^4)^{1/4}\right)}$$

Problem 1216: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$-\frac{(a-bx^4)^{3/4}}{10ax^{10}} - \frac{7b(a-bx^4)^{3/4}}{60a^2x^6} - \frac{7b^2(a-bx^4)^{3/4}}{40a^3x^2} - \frac{7b^{5/2} \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{40a^{5/2}(a-bx^4)^{1/4}}$$

Result (type 5, 95 leaves):

$$\left(-24 a^3 - 4 a^2 b x^4 - 14 a b^2 x^8 + 42 b^3 x^{12} - 21 b^3 x^{12} \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{b x^4}{a} \right] \right) / \left(240 a^3 x^{10} (a - b x^4)^{1/4} \right)$$

Problem 1225: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$\frac{7 a^2 (a - b x^4)^{3/4}}{40 b^3 x} - \frac{7 a x^3 (a - b x^4)^{3/4}}{60 b^2} - \frac{x^7 (a - b x^4)^{3/4}}{10 b} + \frac{7 a^{5/2} \left(1 - \frac{a}{b x^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCsc} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{40 b^{5/2} (a - b x^4)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{60 b^2 (a - b x^4)^{1/4}} x^3 \left(-7 a^2 + a b x^4 + 6 b^2 x^8 + 7 a^2 \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a} \right] \right)$$

Problem 1226: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{a (a - b x^4)^{3/4}}{4 b^2 x} - \frac{x^3 (a - b x^4)^{3/4}}{6 b} + \frac{a^{3/2} \left(1 - \frac{a}{b x^4} \right)^{1/4} x \text{EllipticE} \left[\frac{1}{2} \text{ArcCsc} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{4 b^{3/2} (a - b x^4)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^3 \left(-a + b x^4 + a \left(1 - \frac{b x^4}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a} \right] \right)}{6 b (a - b x^4)^{1/4}}$$

Problem 1227: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a - b x^4)^{1/4}} dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{(a-bx^4)^{3/4}}{2bx} + \frac{\sqrt{a}\left(1-\frac{a}{bx^4}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{2\sqrt{b}(a-bx^4)^{1/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^3 \left(\frac{a-bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^4}{a}\right]}{3(a-bx^4)^{1/4}}$$

Problem 1228: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 61 leaves, 4 steps):

$$-\frac{\sqrt{b}\left(1-\frac{a}{bx^4}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{a}(a-bx^4)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-3a + 3bx^4 - 2bx^4 \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^4}{a}\right]}{3ax(a-bx^4)^{1/4}}$$

Problem 1229: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^6 (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{(a-bx^4)^{3/4}}{5ax^5} - \frac{2b^{3/2}\left(1-\frac{a}{bx^4}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{5a^{3/2}(a-bx^4)^{1/4}}$$

Result (type 5, 84 leaves):

$$\left(-3(a^2 + abx^4 - 2b^2x^8) - 4b^2x^8 \left(1 - \frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^4}{a}\right]\right) / (15a^2x^5(a-bx^4)^{1/4})$$

Problem 1230: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{10} (a-bx^4)^{1/4}} dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{(a-bx^4)^{3/4}}{9ax^9} - \frac{2b(a-bx^4)^{3/4}}{15a^2x^5} - \frac{4b^{5/2}\left(1-\frac{a}{bx^4}\right)^{1/4} \times \text{EllipticE}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{15a^{5/2}(a-bx^4)^{1/4}}$$

Result (type 5, 95 leaves):

$$\left(-5a^3 - a^2bx^4 - 6ab^2x^8 + 12b^3x^{12} - 8b^3x^{12}\left(1-\frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^4}{a}\right]\right) / (45a^3x^9(a-bx^4)^{1/4})$$

Problem 1231: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{14}(a-bx^4)^{1/4}} dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$-\frac{(a-bx^4)^{3/4}}{13ax^{13}} - \frac{10b(a-bx^4)^{3/4}}{117a^2x^9} - \frac{4b^2(a-bx^4)^{3/4}}{39a^3x^5} - \frac{8b^{7/2}\left(1-\frac{a}{bx^4}\right)^{1/4} \times \text{EllipticE}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{39a^{7/2}(a-bx^4)^{1/4}}$$

Result (type 5, 106 leaves):

$$\left(-9a^4 - a^3bx^4 - 2a^2b^2x^8 - 12ab^3x^{12} + 24b^4x^{16} - 16b^4x^{16}\left(1-\frac{bx^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^4}{a}\right]\right) / (117a^4x^{13}(a-bx^4)^{1/4})$$

Problem 1237: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x(a-bx^4)^{3/4}} dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{3/4}} - \frac{\text{ArcTanh}\left[\frac{(a-bx^4)^{1/4}}{a^{1/4}}\right]}{2a^{3/4}}$$

Result (type 5, 49 leaves):

$$-\frac{\left(1-\frac{a}{bx^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{bx^4}\right]}{3(a-bx^4)^{3/4}}$$

Problem 1238: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (a - b x^4)^{3/4}} dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{(a - b x^4)^{1/4}}{4 a x^4} - \frac{3 b \operatorname{ArcTan}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{7/4}} - \frac{3 b \operatorname{ArcTanh}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{8 a^{7/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a + b x^4 - b \left(1 - \frac{a}{b x^4}\right)^{3/4} x^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{b x^4}\right]}{4 a x^4 (a - b x^4)^{3/4}}$$

Problem 1239: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9 (a - b x^4)^{3/4}} dx$$

Optimal (type 3, 108 leaves, 7 steps):

$$-\frac{(a - b x^4)^{1/4}}{8 a x^8} - \frac{7 b (a - b x^4)^{1/4}}{32 a^2 x^4} - \frac{21 b^2 \operatorname{ArcTan}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{11/4}} - \frac{21 b^2 \operatorname{ArcTanh}\left[\frac{(a - b x^4)^{1/4}}{a^{1/4}}\right]}{64 a^{11/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(-4 a^2 - 3 a b x^4 + 7 b^2 x^8 - 7 b^2 \left(1 - \frac{a}{b x^4}\right)^{3/4} x^8 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{a}{b x^4}\right]\right)}{\left(32 a^2 x^8 (a - b x^4)^{3/4}\right)}$$

Problem 1240: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\frac{20 a^2 x^2 (a - b x^4)^{1/4}}{77 b^3} - \frac{10 a x^6 (a - b x^4)^{1/4}}{77 b^2} - \frac{x^{10} (a - b x^4)^{1/4}}{11 b} + \frac{40 a^{7/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{77 b^{7/2} (a - b x^4)^{3/4}}$$

Result (type 5, 92 leaves):

$$\frac{1}{77 b^3 (a - b x^4)^{3/4}} x^2 \left(-20 a^3 + 10 a^2 b x^4 + 3 a b^2 x^8 + 7 b^3 x^{12} + 20 a^3 \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a} \right] \right)$$

Problem 1241: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{2 a x^2 (a - b x^4)^{1/4}}{7 b^2} - \frac{x^6 (a - b x^4)^{1/4}}{7 b} + \frac{4 a^{5/2} \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{7 b^{5/2} (a - b x^4)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{1}{7 b^2 (a - b x^4)^{3/4}} x^2 \left(-2 a^2 + a b x^4 + b^2 x^8 + 2 a^2 \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a} \right] \right)$$

Problem 1242: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{x^2 (a - b x^4)^{1/4}}{3 b} + \frac{2 a^{3/2} \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{3 b^{3/2} (a - b x^4)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^2 \left(-a + b x^4 + a \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a} \right] \right)}{3 b (a - b x^4)^{3/4}}$$

Problem 1243: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{\sqrt{a} \left(1 - \frac{b x^4}{a} \right)^{3/4} \text{EllipticF} \left[\frac{1}{2} \text{ArcSin} \left[\frac{\sqrt{b} x^2}{\sqrt{a}} \right], 2 \right]}{\sqrt{b} (a - b x^4)^{3/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^2 \left(\frac{a-bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^4}{a}\right]}{2 (a-bx^4)^{3/4}}$$

Problem 1244: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{(a-bx^4)^{1/4}}{2ax^2} + \frac{\sqrt{b} \left(1 - \frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{2\sqrt{a} (a-bx^4)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-2a + 2bx^4 + bx^4 \left(1 - \frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^4}{a}\right]}{4ax^2 (a-bx^4)^{3/4}}$$

Problem 1245: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 (a-bx^4)^{3/4}} dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{(a-bx^4)^{1/4}}{6ax^6} - \frac{5b(a-bx^4)^{1/4}}{12a^2x^2} + \frac{5b^{3/2} \left(1 - \frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{12a^{3/2} (a-bx^4)^{3/4}}$$

Result (type 5, 84 leaves):

$$\left(-4a^2 - 6abx^4 + 10b^2x^8 + 5b^2x^8 \left(1 - \frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^4}{a}\right] \right) / \left(24a^2x^6 (a-bx^4)^{3/4} \right)$$

Problem 1246: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} (a-bx^4)^{3/4}} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$-\frac{(a-bx^4)^{1/4}}{10ax^{10}} - \frac{3b(a-bx^4)^{1/4}}{20a^2x^6} - \frac{3b^2(a-bx^4)^{1/4}}{8a^3x^2} + \frac{3b^{5/2} \left(1 - \frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{8a^{5/2} (a-bx^4)^{3/4}}$$

Result (type 5, 95 leaves):

$$\frac{\left(-8 a^3 - 4 a^2 b x^4 - 18 a b^2 x^8 + 30 b^3 x^{12} + 15 b^3 x^{12} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]\right)}{\left(80 a^3 x^{10} (a - b x^4)^{3/4}\right)}$$

Problem 1247: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(a - b x^4)^{3/4}} dx$$

Optimal (type 3, 266 leaves, 12 steps):

$$\begin{aligned} & -\frac{7 a x^3 (a - b x^4)^{1/4}}{32 b^2} - \frac{x^7 (a - b x^4)^{1/4}}{8 b} - \frac{21 a^2 \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{64 \sqrt{2} b^{11/4}} + \\ & \frac{21 a^2 \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{64 \sqrt{2} b^{11/4}} + \frac{21 a^2 \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{128 \sqrt{2} b^{11/4}} - \frac{21 a^2 \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{128 \sqrt{2} b^{11/4}} \end{aligned}$$

Result (type 5, 81 leaves):

$$\frac{1}{32 b^2 (a - b x^4)^{3/4}} x^3 \left(-7 a^2 + 3 a b x^4 + 4 b^2 x^8 + 7 a^2 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]\right)$$

Problem 1248: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a - b x^4)^{3/4}} dx$$

Optimal (type 3, 235 leaves, 11 steps):

$$\begin{aligned} & -\frac{x^3 (a - b x^4)^{1/4}}{4 b} - \frac{3 a \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{8 \sqrt{2} b^{7/4}} + \frac{3 a \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{8 \sqrt{2} b^{7/4}} + \\ & \frac{3 a \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{16 \sqrt{2} b^{7/4}} - \frac{3 a \text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{16 \sqrt{2} b^{7/4}} \end{aligned}$$

Result (type 5, 66 leaves):

$$\frac{x^3 \left(-a + b x^4 + a \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]\right)}{4 b (a - b x^4)^{3/4}}$$

Problem 1249: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a - b x^4)^{3/4}} dx$$

Optimal (type 3, 209 leaves, 10 steps):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{2 \sqrt{2} b^{3/4}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{2 \sqrt{2} b^{3/4}} + \\ & \frac{\text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} - \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{4 \sqrt{2} b^{3/4}} - \frac{\text{Log}\left[1 + \frac{\sqrt{b} x^2}{\sqrt{a - b x^4}} + \frac{\sqrt{2} b^{1/4} x}{(a - b x^4)^{1/4}}\right]}{4 \sqrt{2} b^{3/4}} \end{aligned}$$

Result (type 5, 53 leaves):

$$\frac{x^3 \left(\frac{a - b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]}{3 (a - b x^4)^{3/4}}$$

Problem 1254: Result unnecessarily involves higher level functions.

$$\int \frac{x^{12}}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$\begin{aligned} & -\frac{3 a^2 x (a - b x^4)^{1/4}}{8 b^3} - \frac{3 a x^5 (a - b x^4)^{1/4}}{20 b^2} - \\ & \frac{x^9 (a - b x^4)^{1/4}}{10 b} - \frac{3 a^{5/2} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{8 b^{5/2} (a - b x^4)^{3/4}} \end{aligned}$$

Result (type 5, 91 leaves):

$$\begin{aligned} & \frac{1}{40 b^3 (a - b x^4)^{3/4}} \\ & \left(-15 a^3 x + 9 a^2 b x^5 + 2 a b^2 x^9 + 4 b^3 x^{13} + 15 a^3 x \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]\right) \end{aligned}$$

Problem 1255: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{(a - b x^4)^{3/4}} dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{5 a x (a-b x^4)^{1/4}}{12 b^2} - \frac{x^5 (a-b x^4)^{1/4}}{6 b} - \frac{5 a^{3/2} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 b^{3/2} (a-b x^4)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{12 b^2 (a-b x^4)^{3/4}} \left(-5 a^2 x + 3 a b x^5 + 2 b^2 x^9 + 5 a^2 x \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]\right)$$

Problem 1256: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(a-b x^4)^{3/4}} dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{x (a-b x^4)^{1/4}}{2 b} - \frac{\sqrt{a} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 \sqrt{b} (a-b x^4)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x \left(-a + b x^4 + a \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]\right)}{2 b (a-b x^4)^{3/4}}$$

Problem 1257: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a-b x^4)^{3/4}} dx$$

Optimal (type 4, 63 leaves, 4 steps):

$$-\frac{\sqrt{b} \left(1 - \frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{a} (a-b x^4)^{3/4}}$$

Result (type 5, 48 leaves):

$$\frac{x \left(\frac{a-b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]}{(a-b x^4)^{3/4}}$$

Problem 1258: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a-b x^4)^{3/4}} dx$$

Optimal (type 4, 88 leaves, 5 steps):

$$-\frac{(a-bx^4)^{1/4}}{3ax^3} - \frac{2b^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{3a^{3/2}(a-bx^4)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a+bx^4+2bx^4\left(1-\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^4}{a}\right]}{3ax^3(a-bx^4)^{3/4}}$$

Problem 1259: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8(a-bx^4)^{3/4}} dx$$

Optimal (type 4, 111 leaves, 6 steps):

$$-\frac{(a-bx^4)^{1/4}}{7ax^7} - \frac{2b(a-bx^4)^{1/4}}{7a^2x^3} - \frac{4b^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{7a^{5/2}(a-bx^4)^{3/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(-a^2-abx^4+2b^2x^8+4b^2x^8\left(1-\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^4}{a}\right]\right)}{\left(7a^2x^7(a-bx^4)^{3/4}\right)}$$

Problem 1260: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{12}(a-bx^4)^{3/4}} dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{(a-bx^4)^{1/4}}{11ax^{11}} - \frac{10b(a-bx^4)^{1/4}}{77a^2x^7} - \frac{20b^2(a-bx^4)^{1/4}}{77a^3x^3} - \frac{40b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left[\frac{1}{2}\operatorname{ArcCsc}\left[\frac{\sqrt{b}x^2}{\sqrt{a}}\right], 2\right]}{77a^{7/2}(a-bx^4)^{3/4}}$$

Result (type 5, 95 leaves):

$$\frac{\left(-7a^3-3a^2bx^4-10ab^2x^8+20b^3x^{12}+40b^3x^{12}\left(1-\frac{bx^4}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^4}{a}\right]\right)}{\left(77a^3x^{11}(a-bx^4)^{3/4}\right)}$$

Problem 1261: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a-bx^4)^{5/4}} dx$$

Optimal (type 4, 81 leaves, 5 steps):

$$\frac{1}{b x (a - b x^4)^{1/4}} - \frac{\left(1 - \frac{a}{b x^4}\right)^{1/4} x \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{\sqrt{a} \sqrt{b} (a - b x^4)^{1/4}}$$

Result (type 5, 59 leaves):

$$\frac{x^3 \left(-3 + 2 \left(1 - \frac{b x^4}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]\right)}{3 a (a - b x^4)^{1/4}}$$

Problem 1328: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 (a + b x^6)} dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$-\frac{1}{3 a x^3} - \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} x^3}{\sqrt{a}}\right]}{3 a^{3/2}}$$

Result (type 3, 101 leaves):

$$\frac{1}{3 a^{3/2} x^3} \left(-\sqrt{a} + \sqrt{b} x^3 \operatorname{ArcTan}\left[\frac{b^{1/6} x}{a^{1/6}}\right] + \sqrt{b} x^3 \operatorname{ArcTan}\left[\sqrt{3} - \frac{2 b^{1/6} x}{a^{1/6}}\right] - \sqrt{b} x^3 \operatorname{ArcTan}\left[\sqrt{3} + \frac{2 b^{1/6} x}{a^{1/6}}\right]\right)$$

Problem 1348: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{1 - x^6} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[x^3\right]}{3}$$

Result (type 3, 23 leaves):

$$-\frac{1}{6} \operatorname{Log}\left[1 - x^3\right] + \frac{1}{6} \operatorname{Log}\left[1 + x^3\right]$$

Problem 1394: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{2 + x^6}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\frac{1}{5} x^2 \sqrt{2+x^6} - \left(2 \times 2^{5/6} \sqrt{2+\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(5 \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 133 leaves):

$$\frac{1}{15 \sqrt{2+x^6}} \left(3 x^2 (2+x^6) - 4 (-1)^{1/6} 2^{1/3} \times 3^{3/4} \sqrt{(-1)^{5/6} \left(-1 - \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 1395: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{2+x^6}} dx$$

Optimal (type 4, 166 leaves, 2 steps):

$$\left(\sqrt{2+\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(2^{1/6} \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 116 leaves):

$$\frac{1}{3^{1/4} \sqrt{2+x^6}} (-1)^{1/6} 2^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)}$$

$$\sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 1396: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 \sqrt{2+x^6}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$-\frac{\sqrt{2+x^6}}{8x^4} - \left(\sqrt{2+\sqrt{3}} (2^{1/3} + x^2) \right.$$

$$\left. \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{(2^{1/3} (1 + \sqrt{3}) + x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} (1 - \sqrt{3}) + x^2}{2^{1/3} (1 + \sqrt{3}) + x^2}\right], -7 - 4\sqrt{3}\right] \right) /$$

$$\left(8 \times 2^{1/6} \times 3^{1/4} \sqrt{\frac{2^{1/3} + x^2}{(2^{1/3} (1 + \sqrt{3}) + x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 136 leaves):

$$-\frac{\sqrt{2+x^6}}{8x^4} - \left((-1)^{1/6} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(4 \times 2^{2/3} \times 3^{1/4} \sqrt{2+x^6} \right)$$

Problem 1400: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^9}{\sqrt{2+x^6}} dx$$

Optimal (type 4, 378 leaves, 5 steps):

$$\frac{1}{7} x^4 \sqrt{2+x^6} - \frac{8 \sqrt{2+x^6}}{7 \left(2^{1/3} (1+\sqrt{3}) + x^2\right)} + \left(4 \times 2^{1/6} \times 3^{1/4} \sqrt{2-\sqrt{3}} (2^{1/3} + x^2) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \right. \\ \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], -7-4\sqrt{3}\right]\right) / \left(7 \sqrt{\frac{2^{1/3} + x^2}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \sqrt{2+x^6}\right) - \\ \left(8 \times 2^{2/3} (2^{1/3} + x^2) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], \right. \right. \\ \left. \left. -7-4\sqrt{3}\right]\right) / \left(7 \times 3^{1/4} \sqrt{\frac{2^{1/3} + x^2}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \sqrt{2+x^6}\right)$$

Result (type 4, 189 leaves):

$$\frac{1}{7} x^4 \sqrt{2+x^6} + \frac{1}{7 \times 3^{1/4} \sqrt{2+x^6}} 8 i 2^{2/3} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \\ \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\ \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 1401: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{2+x^6}} dx$$

Optimal (type 4, 354 leaves, 4 steps):

$$\frac{\sqrt{2+x^6}}{2^{1/3} (1+\sqrt{3}) + x^2} - \left(3^{1/4} \sqrt{2-\sqrt{3}} (2^{1/3} + x^2) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{(2^{1/3} (1+\sqrt{3}) + x^2)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], -7-4\sqrt{3}\right] \right) / \left(2^{5/6} \sqrt{\frac{2^{1/3} + x^2}{(2^{1/3} (1+\sqrt{3}) + x^2)^2}} \sqrt{2+x^6} \right) + \left(2^{2/3} (2^{1/3} + x^2) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{(2^{1/3} (1+\sqrt{3}) + x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], -7-4\sqrt{3}\right] \right) / \left(3^{1/4} \sqrt{\frac{2^{1/3} + x^2}{(2^{1/3} (1+\sqrt{3}) + x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 170 leaves):

$$-\frac{1}{3^{1/4} \sqrt{2+x^6}} i^{2^{2/3}} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 1402: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{2+x^6}} dx$$

Optimal (type 4, 378 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{2+x^6}}{4x^2} + \frac{\sqrt{2+x^6}}{4(2^{1/3}(1+\sqrt{3})+x^2)} - \\
 & \left(3^{1/4} \sqrt{2-\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], \right. \right. \\
 & \left. \left. -7-4\sqrt{3}\right] \right) / \left(4 \times 2^{5/6} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right) + \\
 & \left((2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(2 \times 2^{1/3} \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)
 \end{aligned}$$

Result (type 4, 189 leaves):

$$\begin{aligned}
 & -\frac{\sqrt{2+x^6}}{4x^2} - \left(i \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right. \\
 & \left(-i \sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\
 & \left. \left. (-1)^{1/3} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \right) / \left(2 \times 2^{1/3} \times 3^{1/4} \sqrt{2+x^6} \right)
 \end{aligned}$$

Problem 1418: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13}}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{x^8}{3\sqrt{2+x^6}} + \frac{8}{15}x^2\sqrt{2+x^6} - \\
 & \left(16 \times 2^{5/6} \sqrt{2+\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], \right. \right. \\
 & \quad \left. \left. -7-4\sqrt{3}\right] \right) / \left(15 \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)
 \end{aligned}$$

Result (type 4, 144 leaves):

$$\begin{aligned}
 & \frac{1}{45\sqrt{2+x^6}} \left(48x^2 + 9x^8 - 16(-1)^{1/6} 2^{1/3} \times 3^{3/4} \sqrt{-(-1)^{1/6} (2(-1)^{2/3} + 2^{2/3}x^2)} \right. \\
 & \quad \left. \sqrt{2+(-1)^{1/3} 2^{2/3}x^2 + (-1)^{2/3} 2^{1/3}x^4} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] \right)
 \end{aligned}$$

Problem 1419: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{x^2}{3\sqrt{2+x^6}} + \\
 & \left(2^{5/6} \sqrt{2+\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], \right. \right. \\
 & \quad \left. \left. -7-4\sqrt{3}\right] \right) / \left(3 \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)
 \end{aligned}$$

Result (type 4, 136 leaves):

$$-\frac{x^2}{3\sqrt{2+x^6}} + \frac{1}{3 \times 3^{1/4} \sqrt{2+x^6}} 2 (-1)^{1/6} 2^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)}$$

$$\sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 1420: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\frac{x^2}{6\sqrt{2+x^6}} + \left(\sqrt{2+\sqrt{3}} (2^{1/3} + x^2) \right.$$

$$\left. \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{(2^{1/3} (1 + \sqrt{3}) + x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} (1 - \sqrt{3}) + x^2}{2^{1/3} (1 + \sqrt{3}) + x^2}\right], -7 - 4\sqrt{3}\right] \right) /$$

$$\left(6 \times 2^{1/6} \times 3^{1/4} \sqrt{\frac{2^{1/3} + x^2}{(2^{1/3} (1 + \sqrt{3}) + x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 136 leaves):

$$\frac{x^2}{6\sqrt{2+x^6}} + \left((-1)^{1/6} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(3 \times 2^{2/3} \times 3^{1/4} \sqrt{2+x^6} \right)$$

Problem 1421: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5 (2+x^6)^{3/2}} dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\frac{1}{6 x^4 \sqrt{2+x^6}} - \frac{7 \sqrt{2+x^6}}{48 x^4} - \left(7 \sqrt{2+\sqrt{3}} (2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(48 \times 2^{1/6} \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 146 leaves):

$$-\frac{1}{288 x^4 \sqrt{2+x^6}} \left(36 + 42 x^6 + 7 (-1)^{1/6} 2^{1/3} \times 3^{3/4} x^4 \sqrt{-(-1)^{1/6} (2 (-1)^{2/3} + 2^{2/3} x^2)} \sqrt{2 + (-1)^{1/3} 2^{2/3} x^2 + (-1)^{2/3} 2^{1/3} x^4} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(-i + \sqrt{3}) (2 + 2^{2/3} x^2)}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 1426: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{15}}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 394 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{x^{10}}{3\sqrt{2+x^6}} + \frac{10}{21}x^4\sqrt{2+x^6} - \frac{80\sqrt{2+x^6}}{21\left(2^{1/3}(1+\sqrt{3})+x^2\right)} + \\
 & \left(40 \times 2^{1/6}\sqrt{2-\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}}\right. \\
 & \left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right]\right) / \\
 & \left(7 \times 3^{3/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}}\sqrt{2+x^6}\right) - \left(80 \times 2^{2/3}\left(2^{1/3}+x^2\right)\right. \\
 & \left.\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right]\right) / \\
 & \left(21 \times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}}\sqrt{2+x^6}\right)
 \end{aligned}$$

Result (type 4, 195 leaves):

$$\begin{aligned}
 & \frac{1}{63\sqrt{2+x^6}} \left(3x^4(20+3x^6) + \right. \\
 & 40 \times 2^{2/3} \times 3^{3/4} \sqrt{-(-1)^{1/6}\left(2(-1)^{2/3}+2^{2/3}x^2\right)} \sqrt{2+(-1)^{1/3}2^{2/3}x^2+(-1)^{2/3}2^{1/3}x^4} \\
 & \left. \left(\sqrt{3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\
 & \left. \left. (-1)^{5/6}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)
 \end{aligned}$$

Problem 1427: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^9}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 376 leaves, 5 steps):

$$-\frac{x^4}{3\sqrt{2+x^6}} + \frac{4\sqrt{2+x^6}}{3\left(2^{1/3}(1+\sqrt{3})+x^2\right)} - \left(2 \times 2^{1/6} \sqrt{2-\sqrt{3}} \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(3^{3/4} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}} \sqrt{2+x^6}\right) + \left(4 \times 2^{2/3} \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(3 \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}(1+\sqrt{3})+x^2\right)^2}} \sqrt{2+x^6}\right)$$

Result (type 4, 177 leaves):

$$\frac{1}{9\sqrt{2+x^6}} \left(-3x^4 - 4 \times 2^{2/3} \times 3^{3/4} \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right. \\ \left. \left(\sqrt{3} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] + \right. \right. \\ \left. \left. (-1)^{5/6} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{2/3}x^2)}}{2 \times 3^{1/4}}\right], (-1)^{1/3}\right] \right) \right)$$

Problem 1428: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{(2+x^6)^{3/2}} dx$$

Optimal (type 4, 378 leaves, 5 steps):

$$\frac{x^4}{6\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{6(2^{1/3}(1+\sqrt{3})+x^2)} + \left(\sqrt{2-\sqrt{3}}(2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(2 \times 2^{5/6} \times 3^{3/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right) - \left((2^{1/3}+x^2) \sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}(1-\sqrt{3})+x^2}{2^{1/3}(1+\sqrt{3})+x^2}\right], -7-4\sqrt{3}\right] \right) / \left(3 \times 2^{1/3} \times 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{(2^{1/3}(1+\sqrt{3})+x^2)^2}} \sqrt{2+x^6} \right)$$

Result (type 4, 189 leaves):

$$\frac{x^4}{6\sqrt{2+x^6}} + \left(i \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right. \\ \left. - i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right. \\ \left. (-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(3 \times 2^{1/3} \times 3^{1/4} \sqrt{2+x^6} \right)$$

Problem 1429: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 (2+x^6)^{3/2}} dx$$

Optimal (type 4, 394 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{6 x^2 \sqrt{2+x^6}} - \frac{5 \sqrt{2+x^6}}{24 x^2} + \frac{5 \sqrt{2+x^6}}{24 \left(2^{1/3} (1+\sqrt{3}) + x^2\right)} - \\ & \left(5 \sqrt{2-\sqrt{3}} \left(2^{1/3} + x^2\right) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \right. \\ & \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], -7-4\sqrt{3}\right] \right) / \\ & \left(8 \times 2^{5/6} \times 3^{3/4} \sqrt{\frac{2^{1/3} + x^2}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \sqrt{2+x^6} \right) + \\ & \left(5 \left(2^{1/3} + x^2\right) \sqrt{\frac{2^{2/3} - 2^{1/3} x^2 + x^4}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} (1-\sqrt{3}) + x^2}{2^{1/3} (1+\sqrt{3}) + x^2}\right], -7-4\sqrt{3}\right] \right) / \\ & \left(12 \times 2^{1/3} \times 3^{1/4} \sqrt{\frac{2^{1/3} + x^2}{\left(2^{1/3} (1+\sqrt{3}) + x^2\right)^2}} \sqrt{2+x^6} \right) \end{aligned}$$

Result (type 4, 198 leaves):

$$\frac{1}{72 x^2 \sqrt{2+x^6}}$$

$$\int \left(6 i x^6 + 9 i (2+x^6) + 5 i 2^{2/3} \times 3^{3/4} x^2 \sqrt{(-1)^{5/6} \left(-1 + \left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1 + \left(-\frac{1}{2}\right)^{1/3} x^2 + \left(-\frac{1}{2}\right)^{2/3} x^4} \right. \\ \left. \left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-i + \sqrt{3}) (2 + 2^{2/3} x^2)}}{2 \times 3^{1/4}}\right]\right], (-1)^{1/3}\right] + \right. \\ \left. (-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-i + \sqrt{3}) (2 + 2^{2/3} x^2)}}{2 \times 3^{1/4}}\right]\right], (-1)^{1/3}\right] \right)$$

Problem 1456: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 (a + b x^8)} dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$-\frac{1}{4 a x^4} - \frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} x^4}{\sqrt{a}}\right]}{4 a^{3/2}}$$

Result (type 3, 164 leaves):

$$\frac{1}{4 a^{3/2} x^4} \left(-\sqrt{a} + \sqrt{b} x^4 \text{ArcTan}\left[\text{Cot}\left[\frac{\pi}{8}\right] - \frac{b^{1/8} x \text{Csc}\left[\frac{\pi}{8}\right]}{a^{1/8}}\right] + \sqrt{b} x^4 \text{ArcTan}\left[\text{Cot}\left[\frac{\pi}{8}\right] + \frac{b^{1/8} x \text{Csc}\left[\frac{\pi}{8}\right]}{a^{1/8}}\right] + \right. \\ \left. \sqrt{b} x^4 \text{ArcTan}\left[\frac{b^{1/8} x \text{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}} - \text{Tan}\left[\frac{\pi}{8}\right]\right] - \sqrt{b} x^4 \text{ArcTan}\left[\frac{b^{1/8} x \text{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}} + \text{Tan}\left[\frac{\pi}{8}\right]\right] \right)$$

Problem 1472: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{1-x^8} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[x^4\right]}{4}$$

Result (type 3, 23 leaves):

$$-\frac{1}{8} \operatorname{Log}[1-x^4] + \frac{1}{8} \operatorname{Log}[1+x^4]$$

Problem 1494: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (1+x^8)} dx$$

Optimal (type 3, 100 leaves, 11 steps):

$$-\frac{1}{2x^2} + \frac{\operatorname{ArcTan}[1-\sqrt{2}x^2]}{4\sqrt{2}} - \frac{\operatorname{ArcTan}[1+\sqrt{2}x^2]}{4\sqrt{2}} - \frac{\operatorname{Log}[1-\sqrt{2}x^2+x^4]}{8\sqrt{2}} + \frac{\operatorname{Log}[1+\sqrt{2}x^2+x^4]}{8\sqrt{2}}$$

Result (type 3, 208 leaves):

$$\begin{aligned} &-\frac{1}{2x^2} - \frac{\operatorname{ArcTan}\left[\left(x - \cos\left[\frac{\pi}{8}\right]\right) \operatorname{Csc}\left[\frac{\pi}{8}\right]\right]}{4\sqrt{2}} + \frac{\operatorname{ArcTan}\left[\left(x + \cos\left[\frac{\pi}{8}\right]\right) \operatorname{Csc}\left[\frac{\pi}{8}\right]\right]}{4\sqrt{2}} - \\ &\frac{\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{\pi}{8}\right]\left(x - \sin\left[\frac{\pi}{8}\right]\right)\right]}{4\sqrt{2}} + \frac{\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{\pi}{8}\right]\left(x + \sin\left[\frac{\pi}{8}\right]\right)\right]}{4\sqrt{2}} - \frac{\operatorname{Log}\left[1+x^2-2x\cos\left[\frac{\pi}{8}\right]\right]}{8\sqrt{2}} - \\ &\frac{\operatorname{Log}\left[1+x^2+2x\cos\left[\frac{\pi}{8}\right]\right]}{8\sqrt{2}} + \frac{\operatorname{Log}\left[1+x^2-2x\sin\left[\frac{\pi}{8}\right]\right]}{8\sqrt{2}} + \frac{\operatorname{Log}\left[1+x^2+2x\sin\left[\frac{\pi}{8}\right]\right]}{8\sqrt{2}} \end{aligned}$$

Problem 1508: Result unnecessarily involves higher level functions.

$$\int x \sqrt{1+x^8} dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$\frac{1}{6} x^2 \sqrt{1+x^8} + \frac{(1+x^4) \sqrt{\frac{1+x^8}{(1+x^4)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{6 \sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{6} x^2 \left(\sqrt{1+x^8} + 2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right] \right)$$

Problem 1510: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{1+x^8}}{x^3} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$-\frac{\sqrt{1+x^8}}{2x^2} + \frac{x^2\sqrt{1+x^8}}{1+x^4} - \frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{\sqrt{1+x^8}} +$$

$$\frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{2\sqrt{1+x^8}}$$

Result (type 5, 39 leaves):

$$-\frac{\sqrt{1+x^8}}{2x^2} + \frac{1}{3}x^6 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

Problem 1522: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{\sqrt{1+x^8}} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$\frac{1}{10}x^6\sqrt{1+x^8} - \frac{3x^2\sqrt{1+x^8}}{10(1+x^4)} + \frac{3(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{10\sqrt{1+x^8}} -$$

$$\frac{3(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{20\sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{10}x^6\left(\sqrt{1+x^8} - \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]\right)$$

Problem 1523: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\sqrt{1+x^8}} dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$\frac{1}{6}x^2\sqrt{1+x^8} - \frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{12\sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{6}x^2\left(\sqrt{1+x^8} - \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]\right)$$

Problem 1524: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\sqrt{1+x^8}} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$\frac{x^2 \sqrt{1+x^8}}{2(1+x^4)} - \frac{(1+x^4) \sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x^2], \frac{1}{2}\right]}{2 \sqrt{1+x^8}} +$$

$$\frac{(1+x^4) \sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x^2], \frac{1}{2}\right]}{4 \sqrt{1+x^8}}$$

Result (type 5, 22 leaves):

$$\frac{1}{6} x^6 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

Problem 1525: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{1+x^8}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{(1+x^4) \sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x^2], \frac{1}{2}\right]}{4 \sqrt{1+x^8}}$$

Result (type 5, 22 leaves):

$$\frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]$$

Problem 1526: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \sqrt{1+x^8}} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{\sqrt{1+x^8}}{2x^2} + \frac{x^2\sqrt{1+x^8}}{2(1+x^4)} - \frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticE}[2 \text{ArcTan}[x^2], \frac{1}{2}]}{2\sqrt{1+x^8}} +$$

$$\frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}[2 \text{ArcTan}[x^2], \frac{1}{2}]}{4\sqrt{1+x^8}}$$

Result (type 5, 39 leaves):

$$-\frac{\sqrt{1+x^8}}{2x^2} + \frac{1}{6}x^6 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

Problem 1527: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7\sqrt{1+x^8}} dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$-\frac{\sqrt{1+x^8}}{6x^6} - \frac{(1+x^4)\sqrt{\frac{1+x^8}{(1+x^4)^2}} \text{EllipticF}[2 \text{ArcTan}[x^2], \frac{1}{2}]}{12\sqrt{1+x^8}}$$

Result (type 5, 36 leaves):

$$-\frac{\sqrt{1+x^8} + x^8 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]}{6x^6}$$

Problem 1540: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\sqrt{-2+x^{10}}} dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{5} \text{ArcTanh}\left[\frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{10} \text{Log}\left[1 - \frac{x^5}{\sqrt{-2+x^{10}}}\right] + \frac{1}{10} \text{Log}\left[1 + \frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

Problem 1576: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + \frac{b}{x}\right)^3}{x^2} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a + \frac{b}{x}\right)^4}{4b}$$

Result (type 1, 39 leaves):

$$-\frac{b^3}{4x^4} - \frac{ab^2}{x^3} - \frac{3a^2b}{2x^2} - \frac{a^3}{x}$$

Problem 1586: Result more than twice size of optimal antiderivative.

$$\int \left(a + \frac{b}{x}\right)^8 x^{10} dx$$

Optimal (type 1, 47 leaves, 3 steps):

$$\frac{b^2 (b + ax)^9}{9a^3} - \frac{b (b + ax)^{10}}{5a^3} + \frac{(b + ax)^{11}}{11a^3}$$

Result (type 1, 102 leaves):

$$\frac{b^8 x^3}{3} + 2ab^7 x^4 + \frac{28}{5} a^2 b^6 x^5 + \frac{28}{3} a^3 b^5 x^6 + 10a^4 b^4 x^7 + 7a^5 b^3 x^8 + \frac{28}{9} a^6 b^2 x^9 + \frac{4}{5} a^7 b x^{10} + \frac{a^8 x^{11}}{11}$$

Problem 1587: Result more than twice size of optimal antiderivative.

$$\int \left(a + \frac{b}{x}\right)^8 x^9 dx$$

Optimal (type 1, 30 leaves, 3 steps):

$$-\frac{b (b + ax)^9}{9a^2} + \frac{(b + ax)^{10}}{10a^2}$$

Result (type 1, 104 leaves):

$$\frac{b^8 x^2}{2} + \frac{8}{3} a b^7 x^3 + 7a^2 b^6 x^4 + \frac{56}{5} a^3 b^5 x^5 + \frac{35}{3} a^4 b^4 x^6 + 8a^5 b^3 x^7 + \frac{7}{2} a^6 b^2 x^8 + \frac{8}{9} a^7 b x^9 + \frac{a^8 x^{10}}{10}$$

Problem 1598: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + \frac{b}{x}\right)^8}{x^2} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a + \frac{b}{x}\right)^9}{9b}$$

Result (type 1, 96 leaves):

$$-\frac{b^8}{9 x^9} - \frac{a b^7}{x^8} - \frac{4 a^2 b^6}{x^7} - \frac{28 a^3 b^5}{3 x^6} - \frac{14 a^4 b^4}{x^5} - \frac{14 a^5 b^3}{x^4} - \frac{28 a^6 b^2}{3 x^3} - \frac{4 a^7 b}{x^2} - \frac{a^8}{x}$$

Problem 1599: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + \frac{b}{x}\right)^8}{x^3} dx$$

Optimal (type 1, 36 leaves, 3 steps):

$$-\frac{(b + a x)^9}{10 b x^{10}} + \frac{a (b + a x)^9}{90 b^2 x^9}$$

Result (type 1, 104 leaves):

$$-\frac{b^8}{10 x^{10}} - \frac{8 a b^7}{9 x^9} - \frac{7 a^2 b^6}{2 x^8} - \frac{8 a^3 b^5}{x^7} - \frac{35 a^4 b^4}{3 x^6} - \frac{56 a^5 b^3}{5 x^5} - \frac{7 a^6 b^2}{x^4} - \frac{8 a^7 b}{3 x^3} - \frac{a^8}{2 x^2}$$

Problem 1837: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + \frac{b}{x^2}\right)^3}{x^3} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a + \frac{b}{x^2}\right)^4}{8 b}$$

Result (type 1, 43 leaves):

$$-\frac{b^3}{8 x^8} - \frac{a b^2}{2 x^6} - \frac{3 a^2 b}{4 x^4} - \frac{a^3}{2 x^2}$$

Problem 1915: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} x} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 50 leaves):

$$\frac{\sqrt{b + a x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} x}{\sqrt{b + a x^2}}\right]}{\sqrt{a} \sqrt{a + \frac{b}{x^2}} x}$$

Problem 1925: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-a + \frac{b}{x^2}} x} dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-a + \frac{b}{x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 56 leaves):

$$\frac{\sqrt{-b + a x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} x}{\sqrt{-b + a x^2}}\right]}{\sqrt{a} \sqrt{-a + \frac{b}{x^2}} x}$$

Problem 1926: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 + \frac{b}{x^2}} x^2} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$\frac{\operatorname{ArcCsch}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 56 leaves):

$$\frac{\sqrt{b + 2 x^2} \left(\operatorname{Log}[x] - \operatorname{Log}\left[b + \sqrt{b} \sqrt{b + 2 x^2}\right] \right)}{\sqrt{b} \sqrt{2 + \frac{b}{x^2}} x}$$

Problem 1927: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 - \frac{b}{x^2}} x^2} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$\frac{\text{ArcCsc}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 64 leaves):

$$\frac{i \sqrt{2 - \frac{b}{x^2}} x \text{Log}\left[\frac{2(-i\sqrt{b} + \sqrt{-b+2x^2})}{x}\right]}{\sqrt{b} \sqrt{-b+2x^2}}$$

Problem 1957: Result more than twice size of optimal antiderivative.

$$\int \left(1 + \frac{b}{x^2}\right)^{3/2} (c x)^m dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$\frac{(c x)^{1+m} \text{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, -\frac{b}{x^2}\right]}{c(1+m)}$$

Result (type 5, 100 leaves):

$$\left(\sqrt{1 + \frac{b}{x^2}} (c x)^m \left(b m \text{Hypergeometric2F1}\left[-\frac{1}{2}, -1 + \frac{m}{2}, \frac{m}{2}, -\frac{x^2}{b}\right] + \right. \right. \\ \left. \left. (-2+m) x^2 \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{m}{2}, 1 + \frac{m}{2}, -\frac{x^2}{b}\right] \right) \right) / \left((-2+m) m x \sqrt{\frac{b+x^2}{b}} \right)$$

Problem 1960: Result more than twice size of optimal antiderivative.

$$\int \frac{(c x)^m}{\left(1 + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$\frac{(c x)^{1+m} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, -\frac{b}{x^2}\right]}{c(1+m)}$$

Result (type 5, 91 leaves):

$$\frac{1}{(2+m) \sqrt{1+\frac{b}{x^2}}} x (c x)^m \sqrt{\frac{b+x^2}{b}}$$

$$\left(\text{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, 2+\frac{m}{2}, -\frac{x^2}{b}\right] - \text{Hypergeometric2F1}\left[\frac{3}{2}, 1+\frac{m}{2}, 2+\frac{m}{2}, -\frac{x^2}{b}\right] \right)$$

Problem 1996: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} x^7 dx$$

Optimal (type 4, 291 leaves, 5 steps):

$$-\frac{21 b^2 \sqrt{a + \frac{b}{x^3}} x^2}{320 a^2} + \frac{3 b \sqrt{a + \frac{b}{x^3}} x^5}{80 a} + \frac{1}{8} \sqrt{a + \frac{b}{x^3}} x^8 -$$

$$\left(7 \times 3^{3/4} \sqrt{2 + \sqrt{3}} b^{8/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF}\left[\right.$$

$$\left. \text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \left(320 a^2 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 207 leaves):

$$\left(\sqrt{a + \frac{b}{x^3}} x^2 \left((-b)^{1/3} (-21 b^3 - 9 a b^2 x^3 + 52 a^2 b x^6 + 40 a^3 x^9) - \right. \right.$$

$$\left. 7 i 3^{3/4} a^{1/3} b^3 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(320 a^2 (-b)^{1/3} (b + a x^3) \right)$$

Problem 1997: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} x^4 dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\frac{3 b \sqrt{a + \frac{b}{x^3}} x^2}{20 a} + \frac{1}{5} \sqrt{a + \frac{b}{x^3}} x^5 + \left(3^{3/4} \sqrt{2 + \sqrt{3}} b^{5/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \left(20 a \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 196 leaves):

$$\left(\sqrt{a + \frac{b}{x^3}} x^2 \left((-b)^{1/3} (3 b^2 + 7 a b x^3 + 4 a^2 x^6) + i 3^{3/4} a^{1/3} b^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{-(-1)^{5/6} - \frac{i (-b)^{1/3}}{a^{1/3} x}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(20 a (-b)^{1/3} (b + a x^3) \right)$$

Problem 1998: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} x dx$$

Optimal (type 4, 242 leaves, 3 steps):

$$\frac{1}{2} \sqrt{a + \frac{b}{x^3}} x^2 - \left(3^{3/4} \sqrt{2 + \sqrt{3}} b^{2/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\ \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(2 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 162 leaves):

$$\frac{1}{2} \sqrt{a + \frac{b}{x^3}} x^2 \left(1 + \frac{1}{b + a x^3} i 3^{3/4} a^{1/3} (-b)^{2/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x \right. \\ \left. \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 1999: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^2} dx$$

Optimal (type 4, 243 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{a+\frac{b}{x^3}}}{5x} - \left(2 \times 3^{3/4} \sqrt{2+\sqrt{3}} a \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(5 b^{1/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 164 leaves):

$$\begin{aligned}
 & \frac{1}{5x} 2 \sqrt{a+\frac{b}{x^3}} \left(-1 - \left(i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x} \right)} x^4 \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \right. \right. \\
 & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left((-b)^{1/3} (b + a x^3) \right) \right)
 \end{aligned}$$

Problem 2000: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5} dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx} + \\
 & \left(4 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^2 \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\right. \right. \\
 & \left. \left. \operatorname{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7-4\sqrt{3} \right] \right) / \left(55 b^{4/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 192 leaves):

$$\begin{aligned}
 & \left(2 \sqrt{a+\frac{b}{x^3}} \left((-b)^{1/3} (5b^2 + 8abx^3 + 3a^2x^6) - \right. \right. \\
 & \left. \left. 2i3^{3/4} a^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x} \right)} x^7 \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \right. \right. \\
 & \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / (55 (-b)^{4/3} x^4 (b + ax^3))
 \end{aligned}$$

Problem 2001: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8} dx$$

Optimal (type 4, 291 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4} + \frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x} - \\
 & \left(32 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^3 \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(935 b^{7/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 203 leaves):

$$\begin{aligned}
 & \left(2\sqrt{a+\frac{b}{x^3}} \left((-b)^{1/3} (-55b^3 - 70ab^2x^3 + 9a^2bx^6 + 24a^3x^9) - \right. \right. \\
 & 16i3^{3/4}a^{10/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x} \right)} x^{10} \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \\
 & \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / (935(-b)^{7/3}x^7(b+ax^3))
 \end{aligned}$$

Problem 2002: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} x^6 dx$$

Optimal (type 4, 563 leaves, 7 steps):

$$\begin{aligned}
 & \frac{15 b^{7/3} \sqrt{a + \frac{b}{x^3}}}{112 a^2 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{15 b^2 \sqrt{a + \frac{b}{x^3}} x}{112 a^2} + \\
 & \frac{3 b \sqrt{a + \frac{b}{x^3}} x^4}{56 a} + \frac{1}{7} \sqrt{a + \frac{b}{x^3}} x^7 - \left(15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(224 a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(5 \times 3^{3/4} b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(56 \sqrt{2} a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result(type 4, 375 leaves):

$$\frac{1}{112 a^2} \sqrt{a + \frac{b}{x^3}} x \left(-\frac{15 a^{1/3} b^2 x}{b^{1/3} + a^{1/3} x} + 2 a x^3 (3 b + 8 a x^3) - \right.$$

$$\left. \left(15 (-1)^{2/3} b^{7/3} (b^{1/3} + a^{1/3} x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right.$$

$$\left. \left. \left. \left. (1 + i \sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right] \right) \right) \right) /$$

$$\left. \left. \left. \left. \left(2 (-1 + (-1)^{2/3}) (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) \right) \right) \right) \right)$$

Problem 2003: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} x^3 dx$$

Optimal (type 4, 539 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{3 b^{4/3} \sqrt{a + \frac{b}{x^3}}}{8 a \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{3 b \sqrt{a + \frac{b}{x^3}} x}{8 a} + \frac{1}{4} \sqrt{a + \frac{b}{x^3}} x^4 + \\
 & \left(3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(16 a^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(3^{3/4} b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(4 \sqrt{2} a^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 359 leaves):

$$\frac{1}{8} \sqrt{a + \frac{b}{x^3}} x$$

$$\left(2 x^3 + \frac{3 b x}{a^{2/3} b^{1/3} + a x} + \left(3 (-1)^{2/3} b^{4/3} (b^{1/3} + a^{1/3} x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right.$$

$$\left. \left. (1 + i \sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) /$$

$$\left(2 (-1 + (-1)^{2/3}) a (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) \right)$$

Problem 2004: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}} dx$$

Optimal (type 4, 507 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{3 b^{1/3} \sqrt{a + \frac{b}{x^3}}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} + \sqrt{a + \frac{b}{x^3}} x + \\
 & \left(3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{1/3} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticE} \left[\right. \right. \\
 & \left. \left. \operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}, -7 - 4 \sqrt{3} \right] \right) / \left(2 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \\
 & \left(\sqrt{2} 3^{3/4} a^{1/3} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}, -7 - 4 \sqrt{3} \right] \right) / \left(\sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result(type 4, 351 leaves):

$$\int \sqrt{a + \frac{b}{x^3}} dx$$

$$\left(-2 + \frac{3 a^{1/3} x}{b^{1/3} + a^{1/3} x} + \left(3 (-1)^{2/3} b^{1/3} (b^{1/3} + a^{1/3} x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right.$$

$$\left. \left. (1 + i \sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) \Bigg/$$

$$\left(2 (-1 + (-1)^{2/3}) (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) \right)$$

Problem 2005: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^3} dx$$

Optimal (type 4, 517 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{6 a \sqrt{a+\frac{b}{x^3}}}{7 b^{2/3} \left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a+\frac{b}{x^3}}}{7 x^2} + \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(7 b^{2/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(2 \sqrt{2} 3^{3/4} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(7 b^{2/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 366 leaves):

$$\frac{1}{7b} 2 \sqrt{a + \frac{b}{x^3}} x \left(-3a - \frac{b}{x^3} + \frac{3a^{4/3}x}{b^{1/3} + a^{1/3}x} + \right. \\ \left. \left(3(-1)^{2/3} a b^{1/3} (b^{1/3} + a^{1/3}x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3}x (b^{1/3} - (-1)^{1/3} a^{1/3}x)}{(b^{1/3} + a^{1/3}x)^2}} \right. \right. \\ \left. \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3}x}{b^{1/3} + a^{1/3}x}} \left((-3 - i\sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3}x}}{b^{1/3} + a^{1/3}x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right. \right. \\ \left. \left. \left. \left((1 + i\sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3}x}}{b^{1/3} + a^{1/3}x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) \right) \right) / \right. \\ \left. \left. \left. \left(2(-1 + (-1)^{2/3}) (b^{2/3} - a^{1/3} b^{1/3}x + a^{2/3}x^2) \right) \right) \right) \right)$$

Problem 2006: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^6} dx$$

Optimal (type 4, 541 leaves, 6 steps):

$$\begin{aligned}
 & \frac{24 a^2 \sqrt{a + \frac{b}{x^3}}}{91 b^{5/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a + \frac{b}{x^3}}}{13 x^5} - \frac{6 a \sqrt{a + \frac{b}{x^3}}}{91 b x^2} - \\
 & \left(12 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(91 b^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(8 \sqrt{2} 3^{3/4} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(91 b^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 377 leaves):

$$\begin{aligned}
 & \frac{1}{91 b^2} 2 \sqrt{a + \frac{b}{x^3}} x \left(12 a^2 - \frac{7 b^2}{x^6} - \frac{3 a b}{x^3} - \frac{12 a^{7/3} x}{b^{1/3} + a^{1/3} x} \right. \\
 & \left(6 (-1)^{2/3} a^2 b^{1/3} (b^{1/3} + a^{1/3} x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \\
 & \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) + \right. \\
 & \left. \left. \left. \left((1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) \right) \right) / \\
 & \left. \left((-1 + (-1)^{2/3}) (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) \right) \right)
 \end{aligned}$$

Problem 2007: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^9} dx$$

Optimal (type 4, 565 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{240 a^3 \sqrt{a + \frac{b}{x^3}}}{1729 b^{8/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a + \frac{b}{x^3}}}{19 x^8} \\
 & \frac{6 a \sqrt{a + \frac{b}{x^3}}}{247 b x^5} + \frac{60 a^2 \sqrt{a + \frac{b}{x^3}}}{1729 b^2 x^2} + \left(120 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{10/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(80 \sqrt{2} 3^{3/4} a^{10/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result(type 4, 388 leaves):

$$\frac{1}{1729 b^3} 2 \sqrt{a + \frac{b}{x^3}} x \left(-120 a^3 - \frac{91 b^3}{x^9} - \frac{21 a b^2}{x^6} + \frac{30 a^2 b}{x^3} + \frac{120 a^{10/3} x}{b^{1/3} + a^{1/3} x} + \right. \\ \left. \left(60 (-1)^{2/3} a^3 b^{1/3} (b^{1/3} + a^{1/3} x) \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right. \\ \left. \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right. \right. \\ \left. \left. \left. (1 + i \sqrt{3}) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right) \right) / \\ \left. \left((-1 + (-1)^{2/3}) (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) \right) \right)$$

Problem 2017: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{2 \text{ArcTanh} \left[\frac{\sqrt{a + \frac{b}{x^3}}}{\sqrt{a}} \right]}{3 \sqrt{a}}$$

Result (type 3, 59 leaves):

$$\frac{2 \sqrt{b + a x^3} \text{ArcTanh} \left[\frac{\sqrt{a} x^{3/2}}{\sqrt{b + a x^3}} \right]}{3 \sqrt{a} \sqrt{a + \frac{b}{x^3}} x^{3/2}}$$

Problem 2022: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{91 b^2 \sqrt{a + \frac{b}{x^3}} x^2}{320 a^3} - \frac{13 b \sqrt{a + \frac{b}{x^3}} x^5}{80 a^2} + \frac{\sqrt{a + \frac{b}{x^3}} x^8}{8 a} +$$

$$\left(91 \sqrt{2 + \sqrt{3}} b^{8/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(320 \times 3^{1/4} a^3 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 199 leaves):

$$\left(3 (-b)^{1/3} (91 b^3 + 39 a b^2 x^3 - 12 a^2 b x^6 + 40 a^3 x^9) + \right.$$

$$91 i 3^{3/4} a^{1/3} b^3 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}}$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(960 a^3 (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2023: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$-\frac{7b \sqrt{a + \frac{b}{x^3}} x^2}{20a^2} + \frac{\sqrt{a + \frac{b}{x^3}} x^5}{5a}$$

$$\left(7 \sqrt{2 + \sqrt{3}} b^{5/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], \right. \right. \\ \left. \left. -7 - 4\sqrt{3} \right] \right) / \left(20 \times 3^{1/4} a^2 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 188 leaves):

$$\left(-3 (-b)^{1/3} (7b^2 + 3abx^3 - 4a^2x^6) - \right. \\ \left. 7i 3^{3/4} a^{1/3} b^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x} \right)} x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \right. \\ \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}, (-1)^{1/3} \right], (-1)^{1/3} \right] \right) / \left(60 a^2 (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2024: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 248 leaves, 3 steps):

$$\frac{\sqrt{a + \frac{b}{x^3}} x^2}{2a} + \left(\sqrt{2 + \sqrt{3}} b^{2/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4\sqrt{3} \right] \right) / \left(2 \times 3^{1/4} a \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 174 leaves):

$$\frac{b + a x^3}{2a \sqrt{a + \frac{b}{x^3}} x} + \left(i b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} \sqrt{1 + \frac{(-b)^{2/3}}{a^{2/3} x^2} + \frac{(-b)^{1/3}}{a^{1/3} x}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(2 \times 3^{1/4} a^{2/3} (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} \right)$$

Problem 2025: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^2} dx$$

Optimal (type 4, 221 leaves, 2 steps):

$$- \left(2 \sqrt{2 + \sqrt{3}} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4\sqrt{3} \right] \right) / \left(3^{1/4} b^{1/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 142 leaves):

$$- \left(\left(2 i a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} \sqrt{1 + \frac{(-b)^{2/3}}{a^{2/3} x^2} + \frac{(-b)^{1/3}}{a^{1/3} x}} \right. \right. \\ \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(3^{1/4} (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} \right) \right)$$

Problem 2026: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^5} dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$- \frac{2 \sqrt{a + \frac{b}{x^3}}}{5 b x} + \left(4 \sqrt{2 + \sqrt{3}} a \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\ \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(5 \times 3^{1/4} b^{4/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 170 leaves):

$$- \left(\left(-6 (-b)^{1/3} (b + a x^3) + 4 i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x^4 \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \right. \right. \\ \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(15 (-b)^{4/3} \sqrt{a + \frac{b}{x^3}} x^4 \right) \right)$$

Problem 2027: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$-\frac{2\sqrt{a + \frac{b}{x^3}}}{11bx^4} + \frac{16a\sqrt{a + \frac{b}{x^3}}}{55b^2x} - \left(32\sqrt{2 + \sqrt{3}} a^2 \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4\sqrt{3}\right] \right) / \left(55 \times 3^{1/4} b^{7/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 184 leaves):

$$\left(6(-b)^{1/3} (-5b^2 + 3abx^3 + 8a^2x^6) - 32i3^{3/4}a^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x} \right)} x^7 \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(165(-b)^{7/3} \sqrt{a + \frac{b}{x^3}} x^7 \right)$$

Problem 2028: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 566 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{55 b^{7/3} \sqrt{a + \frac{b}{x^3}}}{112 a^3 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{55 b^2 \sqrt{a + \frac{b}{x^3}} x}{112 a^3} - \\
 & \frac{11 b \sqrt{a + \frac{b}{x^3}} x^4}{56 a^2} + \frac{\sqrt{a + \frac{b}{x^3}} x^7}{7 a} + \left(55 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(224 a^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(55 b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(56 \sqrt{2} 3^{1/4} a^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result(type 4, 372 leaves):

$$\left(55 \left(a^{1/3} b^{8/3} x - a^{2/3} b^{7/3} x^2 + a b^2 x^3 \right) + 2 a x^3 \left(-11 b^2 - 3 a b x^3 + 8 a^2 x^6 \right) + \frac{1}{2 \left(-1 + (-1)^{2/3} \right)} \right.$$

$$55 (-1)^{2/3} b^{7/3} \left(b^{1/3} + a^{1/3} x \right)^2 \sqrt{\frac{\left(1 + (-1)^{1/3} \right) a^{1/3} x \left(b^{1/3} - (-1)^{1/3} a^{1/3} x \right)}{\left(b^{1/3} + a^{1/3} x \right)^2}}$$

$$\sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left(-3 - i \sqrt{3} \right) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}, \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) +$$

$$\left. \left(1 + i \sqrt{3} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}, \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right] \right) \Bigg/ \left(112 a^3 \sqrt{a + \frac{b}{x^3}} x^2 \right)$$

Problem 2029: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 542 leaves, 6 steps):

$$\frac{5 b^{4/3} \sqrt{a + \frac{b}{x^3}}}{8 a^2 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{5 b \sqrt{a + \frac{b}{x^3}} x}{8 a^2} + \frac{\sqrt{a + \frac{b}{x^3}} x^4}{4 a} -$$

$$\left(5 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(16 a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(5 b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right.$$

$$\left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(4 \sqrt{2} 3^{1/4} a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 356 leaves):

$$\frac{1}{8 a \sqrt{a + \frac{b}{x^3}} x^2} \left(5 b x \left(-\frac{b^{2/3}}{a^{2/3}} + \frac{b^{1/3} x}{a^{1/3}} - x^2 \right) + 2 x^3 (b + a x^3) - \right.$$

$$\left. \left(5 (-1)^{2/3} b^{4/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right.$$

$$\left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right) \left. \right) / \left(2 (-1 + (-1)^{2/3}) a \right)$$

Problem 2030: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 513 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{b^{1/3} \sqrt{a + \frac{b}{x^3}}}{a \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{\sqrt{a + \frac{b}{x^3}} x}{a} + \\
 & \left(3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticE} \left[\right. \right. \\
 & \left. \left. \operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \left(2 a^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \\
 & \left(\sqrt{2} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], \right. \right. \\
 & \left. \left. -7 - 4 \sqrt{3} \right] \right) / \left(3^{1/4} a^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 334 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{a + \frac{b}{x^3}} x^2} \\
 & \left(x \left(\frac{b^{2/3}}{a^{2/3}} - \frac{b^{1/3} x}{a^{1/3}} + x^2 \right) + \left((-1)^{2/3} b^{1/3} \left(b^{1/3} + a^{1/3} x \right)^2 \sqrt{\frac{\left(1 + (-1)^{1/3} \right) a^{1/3} x \left(b^{1/3} - (-1)^{1/3} a^{1/3} x \right)}{\left(b^{1/3} + a^{1/3} x \right)^2}} \right. \right. \\
 & \left. \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right. \right. \\
 & \left. \left. \left. \left(1 + i \sqrt{3} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right) \right) / \left(2 \left(-1 + (-1)^{2/3} \right) a \right)
 \end{aligned}$$

Problem 2031: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}} dx$$

Optimal (type 4, 491 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{a + \frac{b}{x^3}}}{b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \left(3^{1/4} \sqrt{2 - \sqrt{3}} a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left(b^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(2\sqrt{2} a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left(3^{1/4} b^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 335 leaves):

$$\frac{1}{b \sqrt{a + \frac{b}{x^3}} x^2} \left(-b + a^{1/3} b^{2/3} x - a^{2/3} b^{1/3} x^2 + \frac{1}{2 \left(-1 + (-1)^{2/3} \right)} \right.$$

$$\left. (-1)^{2/3} b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{\left(1 + (-1)^{1/3} \right) a^{1/3} x \left(b^{1/3} - (-1)^{1/3} a^{1/3} x \right)}{\left(b^{1/3} + a^{1/3} x \right)^2}} \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right.$$

$$\left. \left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right] \right) \right)$$

Problem 2032: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^6} dx$$

Optimal (type 4, 520 leaves, 5 steps):

$$\begin{aligned}
 & \frac{8 a \sqrt{a + \frac{b}{x^3}}}{7 b^{5/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a + \frac{b}{x^3}}}{7 b x^2} - \\
 & \left(4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(7 b^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(8 \sqrt{2} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(7 \times 3^{1/4} b^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 363 leaves):

$$\frac{1}{7 b^2 \sqrt{a + \frac{b}{x^3}} x^2}$$

$$2 \left(-4 a^{4/3} x (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) + \frac{(b + a x^3) (-b + 4 a x^3)}{x^3} - \frac{1}{-1 + (-1)^{2/3}} 2 (-1)^{2/3} a b^{1/3} \right.$$

$$\left. (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \right.$$

$$\left((-3 - i \sqrt{3}) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right.$$

$$\left. \left. (1 + i \sqrt{3}) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right)$$

Problem 2033: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^9} dx$$

Optimal (type 4, 544 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{80 a^2 \sqrt{a + \frac{b}{x^3}}}{91 b^{8/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a + \frac{b}{x^3}}}{13 b x^5} + \frac{20 a \sqrt{a + \frac{b}{x^3}}}{91 b^2 x^2} + \\
 & \left(40 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(91 b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(80 \sqrt{2} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(91 \times 3^{1/4} b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 377 leaves):

$$\frac{1}{91 b^3 \sqrt{a + \frac{b}{x^3}} x^2}$$

$$2 \left(40 a^{7/3} x (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) - \frac{(b + a x^3) (7 b^2 - 10 a b x^3 + 40 a^2 x^6)}{x^6} + \frac{1}{-1 + (-1)^{2/3}} \right.$$

$$20 (-1)^{2/3} a^2 b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}}$$

$$\sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right.$$

$$\left. \left. (1 + i \sqrt{3}) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right)$$

Problem 2034: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{12}} dx$$

Optimal (type 4, 568 leaves, 7 steps):

$$\begin{aligned}
 & \frac{1280 a^3 \sqrt{a + \frac{b}{x^3}}}{1729 b^{11/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 \sqrt{a + \frac{b}{x^3}}}{19 b x^8} + \\
 & \frac{32 a \sqrt{a + \frac{b}{x^3}}}{247 b^2 x^5} - \frac{320 a^2 \sqrt{a + \frac{b}{x^3}}}{1729 b^3 x^2} - \left(640 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{10/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 b^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(1280 \sqrt{2} a^{10/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(1729 \times 3^{1/4} b^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result(type 4, 387 leaves):

$$\begin{aligned}
 & 2 \left(-640 a^{10/3} x (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) + \frac{(b + a x^3) (-91 b^3 + 112 a b^2 x^3 - 160 a^2 b x^6 + 640 a^3 x^9)}{x^9} \right. \\
 & \frac{1}{-1 + (-1)^{2/3}} 320 (-1)^{2/3} a^3 b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \\
 & \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \\
 & \left. \left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right]\right] \right) \right) \Bigg/ \left(1729 b^4 \sqrt{a + \frac{b}{x^3}} x^2 \right)
 \end{aligned}$$

Problem 2042: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\left(a + \frac{b}{x^3}\right)^{3/2}} dx$$

Optimal (type 4, 315 leaves, 6 steps):

$$\begin{aligned}
 & \frac{1729 b^2 \sqrt{a + \frac{b}{x^3}} x^2}{960 a^4} - \frac{247 b \sqrt{a + \frac{b}{x^3}} x^5}{240 a^3} \\
 & \frac{2 x^8}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{19 \sqrt{a + \frac{b}{x^3}} x^8}{24 a^2} + \left(1729 \sqrt{2 + \sqrt{3}} b^{8/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4 \sqrt{3}\right] \right) \Bigg/ \\
 & \left(960 \times 3^{1/4} a^4 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 199 leaves):

$$\left(3 (-b)^{1/3} (1729 b^3 + 741 a b^2 x^3 - 228 a^2 b x^6 + 120 a^3 x^9) + \right.$$

$$1729 i 3^{3/4} a^{1/3} b^3 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x}\right)} x \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}}$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(2880 a^4 (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2043: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(a + \frac{b}{x^3}\right)^{3/2}} dx$$

Optimal (type 4, 291 leaves, 5 steps):

$$-\frac{91 b \sqrt{a + \frac{b}{x^3}} x^2}{60 a^3} - \frac{2 x^5}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{13 \sqrt{a + \frac{b}{x^3}} x^5}{15 a^2}$$

$$\left(91 \sqrt{2 + \sqrt{3}} b^{5/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4 \sqrt{3}\right] \right) /$$

$$\left(60 \times 3^{1/4} a^3 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \right)$$

Result (type 4, 188 leaves):

$$\left(-3 (-b)^{1/3} (91 b^2 + 39 a b x^3 - 12 a^2 x^6) - 91 i 3^{3/4} a^{1/3} b^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x}\right)} x \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(180 a^3 (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2044: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(a + \frac{b}{x^3}\right)^{3/2}} dx$$

Optimal (type 4, 269 leaves, 4 steps):

$$-\frac{2 x^2}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{7 \sqrt{a + \frac{b}{x^3}} x^2}{6 a^2} + \left(7 \sqrt{2 + \sqrt{3}} b^{2/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], \right. \right. \\ \left. \left. -7 - 4 \sqrt{3} \right] \right) / \left(6 \times 3^{1/4} a^2 \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \right)$$

Result (type 4, 175 leaves):

$$\left(3 (-b)^{1/3} (7b + 3ax^3) + 7i 3^{3/4} a^{1/3} b \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x}\right)} x \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(18a^2 (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2045: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^2} dx$$

Optimal (type 4, 248 leaves, 3 steps):

$$-\frac{2}{3a \sqrt{a + \frac{b}{x^3}} x} - \left(2 \sqrt{2 + \sqrt{3}} \left(a^{1/3} + \frac{b^{1/3}}{x}\right) \right. \\ \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left((1 + \sqrt{3})a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3})a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3})a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4\sqrt{3}\right] \right) / \\ \left(3 \times 3^{1/4} a b^{1/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left((1 + \sqrt{3})a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} \right)$$

Result (type 4, 164 leaves):

$$\left(-6 (-b)^{1/3} - 2i 3^{3/4} a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}x}\right)} x \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3}x}{a^{1/3}} + x^2}{x^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \left(9a (-b)^{1/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2046: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^5} dx$$

Optimal (type 4, 245 leaves, 3 steps):

$$\frac{2}{3 b \sqrt{a + \frac{b}{x^3}} x} - \left(4 \sqrt{2 + \sqrt{3}} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \left(3 \times 3^{1/4} b^{4/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 161 leaves):

$$- \left(\left(6 (-b)^{1/3} - 4 i 3^{3/4} a^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(9 (-b)^{4/3} \sqrt{a + \frac{b}{x^3}} x \right)$$

Problem 2047: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^8} dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\frac{2}{3 b \sqrt{a + \frac{b}{x^3}} x^4} - \frac{16 \sqrt{a + \frac{b}{x^3}}}{15 b^2 x} +$$

$$\left(32 \sqrt{2 + \sqrt{3}} a \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}\right], \right.$$

$$\left. -7 - 4 \sqrt{3} \right] \Bigg/ \left(15 \times 3^{1/4} b^{7/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 173 leaves):

$$\left(-6 (-b)^{1/3} (3 b + 8 a x^3) + 32 i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3} x} \right)} x^4 \sqrt{\frac{(-b)^{2/3} + \frac{(-b)^{1/3} x}{a^{1/3}} + x^2}{x^2}} \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3}}{a^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) \Bigg/ \left(45 (-b)^{7/3} \sqrt{a + \frac{b}{x^3}} x^4 \right)$$

Problem 2048: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\left(a + \frac{b}{x^3}\right)^{3/2}} dx$$

Optimal (type 4, 587 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{935 b^{7/3} \sqrt{a + \frac{b}{x^3}}}{336 a^4 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{935 b^2 \sqrt{a + \frac{b}{x^3}} x}{336 a^4} - \frac{187 b \sqrt{a + \frac{b}{x^3}} x^4}{168 a^3} - \\
 & \frac{2 x^7}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{17 \sqrt{a + \frac{b}{x^3}} x^7}{21 a^2} + \left(935 \sqrt{2 - \sqrt{3}} b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(224 \times 3^{3/4} a^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} - 935 b^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(168 \sqrt{2} 3^{1/4} a^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 390 leaves):

$$\left((b + a x^3) \right.$$

$$\left(-224 a b^2 x^3 - 150 a b x^3 (b + a x^3) + 48 a^2 x^6 (b + a x^3) + 935 (a^{1/3} b^{8/3} x - a^{2/3} b^{7/3} x^2 + a b^2 x^3) + \right.$$

$$\left. 935 (-1)^{2/3} b^{7/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right.$$

$$\left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) /$$

$$\left. \left. \left(2 (-1 + (-1)^{2/3}) \right) \right) \right) / \left(336 a^4 \left(a + \frac{b}{x^3} \right)^{3/2} x^5 \right)$$

Problem 2049: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\left(a + \frac{b}{x^3}\right)^{3/2}} dx$$

Optimal (type 4, 563 leaves, 7 steps):

$$\frac{55 b^{4/3} \sqrt{a + \frac{b}{x^3}}}{24 a^3 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{55 b \sqrt{a + \frac{b}{x^3}} x}{24 a^3} -$$

$$\frac{2 x^4}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{11 \sqrt{a + \frac{b}{x^3}} x^4}{12 a^2} - \left(55 \sqrt{2 - \sqrt{3}} b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right.$$

$$\left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right/$$

$$\left(16 \times 3^{3/4} a^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(55 b^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right.$$

$$\left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right/$$

$$\left(12 \sqrt{2} 3^{1/4} a^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)$$

Result (type 4, 370 leaves):

$$\left((b + a x^3) \left(16 a b x^3 + 6 a x^3 (b + a x^3) - 55 (a^{1/3} b^{5/3} x - a^{2/3} b^{4/3} x^2 + a b x^3) - \right. \right.$$

$$\left. \left. 55 (-1)^{2/3} b^{4/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right. \right.$$

$$\left. \left. \left. (1 + i \sqrt{3}) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right) \right) /$$

$$\left. \left. \left. \left(2 (-1 + (-1)^{2/3}) \right) \right) \right) / \left(24 a^3 \left(a + \frac{b}{x^3} \right)^{3/2} x^5 \right)$$

Problem 2050: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3} \right)^{3/2}} dx$$

Optimal (type 4, 539 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{5 b^{1/3} \sqrt{a + \frac{b}{x^3}}}{3 a^2 \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2 x}{3 a \sqrt{a + \frac{b}{x^3}}} + \frac{5 \sqrt{a + \frac{b}{x^3}} x}{3 a^2} + \\
 & \left(5 \sqrt{2 - \sqrt{3}} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(2 \times 3^{3/4} a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(5 \sqrt{2} b^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(3 \times 3^{1/4} a^{5/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 353 leaves):

$$\frac{1}{3 a \left(a + \frac{b}{x^3} \right)^{3/2} x^5} \left(b + a x^3 \right) \left(-2 x^3 + 5 x \left(\frac{b^{2/3}}{a^{2/3}} - \frac{b^{1/3} x}{a^{1/3}} + x^2 \right) + \right.$$

$$\left. \left(5 (-1)^{2/3} b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right.$$

$$\left. \left. \left. \left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right] \right) \right) \right) \right) \right) / \left(2 (-1 + (-1)^{2/3}) a \right)$$

Problem 2051: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3} \right)^{3/2} x^3} dx$$

Optimal (type 4, 520 leaves, 5 steps):

$$\frac{2 \sqrt{a + \frac{b}{x^3}}}{3 a b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} - \frac{2}{3 a \sqrt{a + \frac{b}{x^3}} x^2} - \left(\sqrt{2 - \sqrt{3}} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\ \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\ \left(3^{3/4} a^{2/3} b^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} + \left(2 \sqrt{2} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \right. \\ \left. \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \right. \\ \left. \left(3 \times 3^{1/4} a^{2/3} b^{2/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) \right)$$

Result (type 4, 352 leaves):

$$\frac{1}{3 b \left(a + \frac{b}{x^3} \right)^{3/2} x^5} \left(2 (b + a x^3) \left(x^3 + x \left(-\frac{b^{2/3}}{a^{2/3}} + \frac{b^{1/3} x}{a^{1/3}} - x^2 \right) - \right. \right. \\ \left. \left((-1)^{2/3} b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right. \\ \left. \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + \right. \right. \right. \\ \left. \left. \left. (1 + i \sqrt{3}) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right) \right) / \left(2 (-1 + (-1)^{2/3}) a \right)$$

Problem 2052: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^6} dx$$

Optimal (type 4, 517 leaves, 5 steps):

$$\begin{aligned} & -\frac{8\sqrt{a + \frac{b}{x^3}}}{3b^{5/3}\left(\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}\right)} + \frac{2}{3b\sqrt{a + \frac{b}{x^3}}x^2} + \\ & \left(4\sqrt{2 - \sqrt{3}}a^{1/3}\left(a^{1/3} + \frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}\right)^2}}\right. \\ & \left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}}{\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4\sqrt{3}\right]\right) / \\ & \left(3^{3/4}b^{5/3}\sqrt{a + \frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left(\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}\right)^2}} - \left(8\sqrt{2}a^{1/3}\left(a^{1/3} + \frac{b^{1/3}}{x}\right)\right.\right. \\ & \left.\left.\sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}\right)^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}}{\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4\sqrt{3}\right]\right) / \\ & \left(3 \times 3^{1/4}b^{5/3}\sqrt{a + \frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left(\left(1 + \sqrt{3}\right)a^{1/3} + \frac{b^{1/3}}{x}\right)^2}}\right) \end{aligned}$$

Result (type 4, 362 leaves):

$$\frac{1}{3 b^2 \left(a + \frac{b}{x^3}\right)^{3/2} x^5}$$

$$2 (b + a x^3) \left(-a x^3 - 3 (b + a x^3) + 4 (a^{1/3} b^{2/3} x - a^{2/3} b^{1/3} x^2 + a x^3) + \frac{1}{-1 + (-1)^{2/3}} 2 (-1)^{2/3} b^{1/3} \right.$$

$$\left. (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \right.$$

$$\left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right.$$

$$\left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right)$$

Problem 2053: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^9} dx$$

Optimal (type 4, 541 leaves, 6 steps):

$$\begin{aligned}
 & \frac{80 a \sqrt{a + \frac{b}{x^3}}}{21 b^{8/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{2}{3 b \sqrt{a + \frac{b}{x^3}} x^5} - \\
 & \frac{20 \sqrt{a + \frac{b}{x^3}}}{21 b^2 x^2} - \left(40 \sqrt{2 - \sqrt{3}} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(7 \times 3^{3/4} b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) + \left(80 \sqrt{2} a^{4/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(21 \times 3^{1/4} b^{8/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 380 leaves):

$$\left(2 (b + a x^3) \left(7 a^2 x^3 - 40 a^{4/3} x (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) + 33 a (b + a x^3) - \frac{3 b (b + a x^3)}{x^3} - \frac{1}{-1 + (-1)^{2/3}} 20 (-1)^{2/3} a b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \right. \right.$$

$$\left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right.$$

$$\left. \left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(3+i\sqrt{3}) a^{1/3} x}{b^{1/3} + a^{1/3} x}}}{\sqrt{2}}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) \right) / \left(21 b^3 \left(a + \frac{b}{x^3} \right)^{3/2} x^5 \right)$$

Problem 2054: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^{12}} dx$$

Optimal (type 4, 565 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{1280 a^2 \sqrt{a + \frac{b}{x^3}}}{273 b^{11/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)} + \frac{2}{3 b \sqrt{a + \frac{b}{x^3}} x^8} \\
 & \frac{32 \sqrt{a + \frac{b}{x^3}}}{39 b^2 x^5} + \frac{320 a \sqrt{a + \frac{b}{x^3}}}{273 b^3 x^2} + \left(640 \sqrt{2 - \sqrt{3}} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(91 \times 3^{3/4} b^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right) - \left(1280 \sqrt{2} a^{7/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3} b^{1/3}}{x}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 & \left(273 \times 3^{1/4} b^{11/3} \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3}}{x} \right)^2}} \right)
 \end{aligned}$$

Result (type 4, 400 leaves):

$$\begin{aligned}
 & \left(2 (b + a x^3) \left(-91 a^3 x^3 + 640 a^{7/3} x (b^{2/3} - a^{1/3} b^{1/3} x + a^{2/3} x^2) - \right. \right. \\
 & 549 a^2 (b + a x^3) - \frac{21 b^2 (b + a x^3)}{x^6} + \frac{69 a b (b + a x^3)}{x^3} + \frac{1}{-1 + (-1)^{2/3}} \\
 & 320 (-1)^{2/3} a^2 b^{1/3} (b^{1/3} + a^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) a^{1/3} x (b^{1/3} - (-1)^{1/3} a^{1/3} x)}{(b^{1/3} + a^{1/3} x)^2}} \\
 & \left. \sqrt{\frac{b^{1/3} + (-1)^{2/3} a^{1/3} x}{b^{1/3} + a^{1/3} x}} \left((-3 - i \sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + \right. \right. \\
 & \left. \left. (1 + i \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3}) a^{1/3} x}}{b^{1/3} + a^{1/3} x}}{\sqrt{2}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right) \Bigg/ \left(273 b^4 \left(a + \frac{b}{x^3}\right)^{3/2} x^5 \right)
 \end{aligned}$$

Problem 2060: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^4}} x^2 dx$$

Optimal (type 4, 107 leaves, 3 steps):

$$\frac{1}{3} \sqrt{a + \frac{b}{x^4}} x^3 - \frac{b^{3/4} \sqrt{\frac{a + \frac{b}{x^4}}{(\sqrt{a} + \frac{\sqrt{b}}{x^2})^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{3 a^{1/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 93 leaves):

$$\frac{1}{3} \sqrt{a + \frac{b}{x^4}} x^2 \left(x - \frac{2 i b \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (b + a x^4)} \right)$$

Problem 2061: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^4}} dx$$

Optimal (type 4, 224 leaves, 5 steps):

$$\frac{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)x} + \sqrt{a+\frac{b}{x^4}}x + \frac{2a^{1/4}b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)^2}}\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)\text{EllipticE}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{a+\frac{b}{x^4}}}$$

$$\frac{a^{1/4}b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)^2}}\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)\text{EllipticF}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 119 leaves):

$$\sqrt{a+\frac{b}{x^4}}x\left(-1+\left(2i\sqrt{a}\sqrt{1+\frac{ax^4}{b}}\left(\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x\right], -1\right]-\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x\right], -1\right]\right)\right)/\left(\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}(b+ax^4)\right)\right)$$

Problem 2062: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^4}}}{x^2} dx$$

Optimal (type 4, 107 leaves, 3 steps):

$$\frac{\sqrt{a+\frac{b}{x^4}}}{3x} - \frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)^2}}\left(\sqrt{a+\frac{\sqrt{b}}{x^2}}\right)\text{EllipticF}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{3b^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 96 leaves):

$$\frac{\sqrt{a + \frac{b}{x^4}}}{3x} \left(-1 - \frac{2 i a x^3 \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (b + a x^4)} \right)$$

Problem 2063: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{x^4}}}{x^4} dx$$

Optimal (type 4, 236 leaves, 5 steps):

$$\frac{\sqrt{a + \frac{b}{x^4}}}{5x^3} - \frac{2a\sqrt{a + \frac{b}{x^4}}}{5\sqrt{b}\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)x} + \frac{2a^{5/4}\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticE}\left[2\operatorname{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{3/4}\sqrt{a + \frac{b}{x^4}}}$$

$$\frac{a^{5/4}\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left[2\operatorname{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{3/4}\sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 138 leaves):

$$\frac{1}{5}\sqrt{a + \frac{b}{x^4}}x^2 \left(-\frac{b + 2ax^4}{bx^5} - \frac{1}{b + ax^4} 2i a \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} \sqrt{1 + \frac{ax^4}{b}} \right. \\ \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right)$$

Problem 2068: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + \frac{b}{x^4}\right)^{3/2} x^2 dx$$

Optimal (type 4, 126 leaves, 4 steps):

$$-\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{3/2}x^3 - \frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^4}}\right)^2}}\left(\sqrt{a+\frac{b}{x^4}}\right)\text{EllipticF}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{3\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 128 leaves):

$$\left(\sqrt{a+\frac{b}{x^4}}\right) \left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-b^2+a^2x^8\right)-4iabx^3\sqrt{1+\frac{ax^4}{b}}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right]x, -1\right]\right) \Big/ \left(3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right) x\left(b+ax^4\right)$$

Problem 2069: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx$$

Optimal (type 4, 250 leaves, 6 steps):

$$-\frac{6b\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{12a\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{5\left(\sqrt{a+\frac{b}{x^4}}\right)x} + \left(a+\frac{b}{x^4}\right)^{3/2}x + \frac{12a^{5/4}b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^4}}\right)^2}}\left(\sqrt{a+\frac{b}{x^4}}\right)\text{EllipticE}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{5\sqrt{a+\frac{b}{x^4}}} - \frac{6a^{5/4}b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^4}}\right)^2}}\left(\sqrt{a+\frac{b}{x^4}}\right)\text{EllipticF}\left[2\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right], \frac{1}{2}\right]}{5\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 196 leaves):

$$\begin{aligned}
 & - \left(\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (b^2 + 8 a b x^4 + 7 a^2 x^8) - \right. \right. \right. \\
 & \quad \left. \left. \left. 12 a^{3/2} \sqrt{b} x^5 \sqrt{1 + \frac{a x^4}{b}} \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] + 12 a^{3/2} \sqrt{b} x^5 \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) \right) \right) / \left(5 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^3 (b + a x^4) \right)
 \end{aligned}$$

Problem 2070: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 126 leaves, 4 steps):

$$\frac{2 a \sqrt{a + \frac{b}{x^4}}}{7 x} - \frac{\left(a + \frac{b}{x^4}\right)^{3/2}}{7 x} - \frac{2 a^{7/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF} \left[2 \text{ArcCot} \left[\frac{a^{1/4} x}{b^{1/4}} \right], \frac{1}{2} \right]}{7 b^{1/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 135 leaves):

$$\begin{aligned}
 & - \left(\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (b^2 + 4 a b x^4 + 3 a^2 x^8) + \right. \right. \right. \\
 & \quad \left. \left. \left. 4 i a^2 x^7 \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) \right) \right) / \left(7 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^5 (b + a x^4) \right)
 \end{aligned}$$

Problem 2071: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 257 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{2 a \sqrt{a+\frac{b}{x^4}}}{15 x^3}-\frac{\left(a+\frac{b}{x^4}\right)^{3 / 2}}{9 x^3}-\frac{4 a^2 \sqrt{a+\frac{b}{x^4}}}{15 \sqrt{b}\left(\sqrt{a+\frac{b}{x^2}}\right) x}+ \\
 & \frac{4 a^{9 / 4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^2}}\right)^2}}\left(\sqrt{a+\frac{b}{x^2}}\right) \operatorname{EllipticE}\left[2 \operatorname{ArcCot}\left[\frac{a^{1 / 4} x}{b^{1 / 4}}\right], \frac{1}{2}\right]}{15 b^{3 / 4} \sqrt{a+\frac{b}{x^4}}}- \\
 & \frac{2 a^{9 / 4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^2}}\right)^2}}\left(\sqrt{a+\frac{b}{x^2}}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{1 / 4} x}{b^{1 / 4}}\right], \frac{1}{2}\right]}{15 b^{3 / 4} \sqrt{a+\frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
 & \frac{\left(a+\frac{b}{x^4}\right)^{3 / 2}\left(-\frac{b}{9 x^9}-\frac{11 a}{45 x^5}-\frac{4 a^2}{15 b x}\right) x^6}{b+a x^4}+ \\
 & \left(4 a^{5 / 2}\left(a+\frac{b}{x^4}\right)^{3 / 2} x^6 \sqrt{1-\frac{i \sqrt{a} x^2}{\sqrt{b}}}\sqrt{1+\frac{i \sqrt{a} x^2}{\sqrt{b}}}\left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\right] x\right],-1\right)-\right. \\
 & \left.\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\right] x\right],-1\right)\right) / \left(15 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\sqrt{b}\left(b+a x^4\right)^2\right)
 \end{aligned}$$

Problem 2076: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+\frac{b}{x^4}\right)^{5 / 2} x^2 d x$$

Optimal (type 4, 146 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{20 a b \sqrt{a+\frac{b}{x^4}}}{21 x}-\frac{10 b\left(a+\frac{b}{x^4}\right)^{3 / 2}}{21 x}+\frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5 / 2} x^3- \\
 & \frac{20 a^{7 / 4} b^{3 / 4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a+\frac{b}{x^2}}\right)^2}}\left(\sqrt{a+\frac{b}{x^2}}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{1 / 4} x}{b^{1 / 4}}\right], \frac{1}{2}\right]}{21 \sqrt{a+\frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 149 leaves):

$$\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (-3 b^3 - 19 a b^2 x^4 - 9 a^2 b x^8 + 7 a^3 x^{12}) - 40 i a^2 b x^7 \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) \right) / \left(21 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^5 (b + a x^4) \right)$$

Problem 2077: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + \frac{b}{x^4} \right)^{5/2} dx$$

Optimal (type 4, 272 leaves, 7 steps):

$$\begin{aligned} & -\frac{4 a b \sqrt{a + \frac{b}{x^4}}}{3 x^3} - \frac{10 b \left(a + \frac{b}{x^4} \right)^{3/2}}{9 x^3} - \frac{8 a^2 \sqrt{b} \sqrt{a + \frac{b}{x^4}}}{3 \left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right) x} + \left(a + \frac{b}{x^4} \right)^{5/2} x + \\ & \frac{8 a^{9/4} b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right) \operatorname{EllipticE} \left[2 \operatorname{ArcCot} \left[\frac{a^{1/4} x}{b^{1/4}} \right], \frac{1}{2} \right]}{3 \sqrt{a + \frac{b}{x^4}}} + \\ & \frac{4 a^{9/4} b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right) \operatorname{EllipticF} \left[2 \operatorname{ArcCot} \left[\frac{a^{1/4} x}{b^{1/4}} \right], \frac{1}{2} \right]}{3 \sqrt{a + \frac{b}{x^4}}} \end{aligned}$$

Result (type 4, 207 leaves):

$$\begin{aligned} & - \left(\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (b^3 + 5 a b^2 x^4 + 19 a^2 b x^8 + 15 a^3 x^{12}) - 24 a^{5/2} \sqrt{b} x^9 \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] + 24 a^{5/2} \sqrt{b} x^9 \right. \right. \right. \\ & \left. \left. \left. \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) \right) \right) / \left(9 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^7 (b + a x^4) \right) \end{aligned}$$

Problem 2078: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{5/2}}{x^2} dx$$

Optimal (type 4, 147 leaves, 5 steps):

$$\frac{20 a^2 \sqrt{a + \frac{b}{x^4}}}{77 x} - \frac{10 a \left(a + \frac{b}{x^4}\right)^{3/2}}{77 x} - \frac{\left(a + \frac{b}{x^4}\right)^{5/2}}{11 x} - \frac{20 a^{11/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{77 b^{1/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 148 leaves):

$$\frac{\left(\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (7 b^3 + 31 a b^2 x^4 + 61 a^2 b x^8 + 37 a^3 x^{12}) + 40 i a^3 x^{11} \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]\right)\right)\right)}{\left(77 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^9 (b + a x^4)\right)}$$

Problem 2079: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{x^4}\right)^{5/2}}{x^4} dx$$

Optimal (type 4, 278 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{4 a^2 \sqrt{a + \frac{b}{x^4}}}{39 x^3} - \frac{10 a \left(a + \frac{b}{x^4}\right)^{3/2}}{117 x^3} - \frac{\left(a + \frac{b}{x^4}\right)^{5/2}}{13 x^3} - \frac{8 a^3 \sqrt{a + \frac{b}{x^4}}}{39 \sqrt{b} \left(\sqrt{a + \frac{b}{x^4}}\right) x} + \\
 & \frac{8 a^{13/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^4}}\right)^2}} \left(\sqrt{a + \frac{b}{x^4}}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{39 b^{3/4} \sqrt{a + \frac{b}{x^4}}} - \\
 & \frac{4 a^{13/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^4}}\right)^2}} \left(\sqrt{a + \frac{b}{x^4}}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{39 b^{3/4} \sqrt{a + \frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 223 leaves):

$$\begin{aligned}
 & - \left(\left(\sqrt{a + \frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \left(9 b^4 + 37 a b^3 x^4 + 59 a^2 b^2 x^8 + 55 a^3 b x^{12} + 24 a^4 x^{16} \right) - \right. \right. \right. \\
 & \quad \left. \left. \left. 24 a^{7/2} \sqrt{b} x^{13} \sqrt{1 + \frac{a x^4}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] + 24 a^{7/2} \sqrt{b} x^{13} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right) \right) / \left(117 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} b x^{11} (b + a x^4) \right)
 \end{aligned}$$

Problem 2082: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}} x} dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a + \frac{b}{x^4}}}{\sqrt{a}}\right]}{2 \sqrt{a}}$$

Result (type 3, 55 leaves):

$$\frac{\sqrt{b + a x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{a} x^2}{\sqrt{b + a x^4}}\right]}{2 \sqrt{a} \sqrt{a + \frac{b}{x^4}} x^2}$$

Problem 2084: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + \frac{b}{x^4}}} dx$$

Optimal (type 4, 110 leaves, 3 steps):

$$\frac{\sqrt{a + \frac{b}{x^4}} x^3}{3 a} + \frac{b^{3/4} \sqrt{\frac{a + \frac{b}{x^2}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{6 a^{5/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 113 leaves):

$$\left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x (b + a x^4) + i b \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) / \left(3 a \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{a + \frac{b}{x^4}} x^2 \right)$$

Problem 2085: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}}} dx$$

Optimal (type 4, 231 leaves, 5 steps):

$$\frac{\sqrt{b} \sqrt{a + \frac{b}{x^4}}}{a \left(\sqrt{a + \frac{b}{x^2}}\right) x} + \frac{\sqrt{a + \frac{b}{x^4}} x}{a} + \frac{b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a + \frac{b}{x^2}}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} \sqrt{a + \frac{b}{x^4}}}$$

$$\frac{b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a + \frac{b}{x^2}}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 107 leaves):

$$\left(i \sqrt{1 + \frac{a x^4}{b}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right) / \left(\left(\frac{i \sqrt{a}}{\sqrt{b}} \right)^{3/2} \sqrt{a + \frac{b}{x^4}} x^2 \right)$$

Problem 2086: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}} x^2} dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$\frac{\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a + \frac{b}{x^2}}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 77 leaves):

$$\frac{i \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{a + \frac{b}{x^4}} x^2}$$

Problem 2087: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}}} dx$$

Optimal (type 4, 212 leaves, 4 steps):

$$-\frac{\sqrt{a + \frac{b}{x^4}}}{\sqrt{b} \left(\sqrt{a + \frac{b}{x^2}} \right) x} + \frac{a^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}} \right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right) \text{EllipticE} \left[2 \text{ArcCot} \left[\frac{a^{1/4} x}{b^{1/4}} \right], \frac{1}{2} \right]}{b^{3/4} \sqrt{a + \frac{b}{x^4}}}$$

$$\frac{a^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}} \right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2} \right) \text{EllipticF} \left[2 \text{ArcCot} \left[\frac{a^{1/4} x}{b^{1/4}} \right], \frac{1}{2} \right]}{2 b^{3/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 173 leaves):

$$-\frac{b + a x^4}{b \sqrt{a + \frac{b}{x^4}} x^3} + \left(\sqrt{a} \sqrt{1 - \frac{i \sqrt{a} x^2}{\sqrt{b}}} \sqrt{1 + \frac{i \sqrt{a} x^2}{\sqrt{b}}} \right. \\ \left. \left(\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] - \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) \right) / \\ \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{b} \sqrt{a + \frac{b}{x^4}} x^2 \right)$$

Problem 2092: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(a + \frac{b}{x^4} \right)^{3/2}} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$-\frac{x^3}{2 a \sqrt{a + \frac{b}{x^4}}} + \frac{5 \sqrt{a + \frac{b}{x^4}} x^3}{6 a^2} + \frac{5 b^{3/4} \sqrt{\frac{a + \frac{b}{x^4}}{(\sqrt{a} + \frac{\sqrt{b}}{x^2})^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{12 a^{9/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 116 leaves):

$$\left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x (5 b + 2 a x^4) + 5 i b \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) / \left(6 a^2 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{a + \frac{b}{x^4}} x^2 \right)$$

Problem 2093: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{3/2}} dx$$

Optimal (type 4, 258 leaves, 6 steps):

$$-\frac{3 \sqrt{b} \sqrt{a + \frac{b}{x^4}}}{2 a^2 \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) x} - \frac{x}{2 a \sqrt{a + \frac{b}{x^4}}} + \frac{3 \sqrt{a + \frac{b}{x^4}} x}{2 a^2} + \frac{3 b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{(\sqrt{a} + \frac{\sqrt{b}}{x^2})^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{2 a^{7/4} \sqrt{a + \frac{b}{x^4}}} - \frac{3 b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{(\sqrt{a} + \frac{\sqrt{b}}{x^2})^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{7/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 166 leaves):

$$\left(-\sqrt{a} \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x^3 + 3\sqrt{b} \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x \right], -1 \right] - \right. \\ \left. 3\sqrt{b} \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x \right], -1 \right] \right) / \left(2 a^{3/2} \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} \sqrt{a + \frac{b}{x^4}} x^2 \right)$$

Problem 2094: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{3/2} x^2} dx$$

Optimal (type 4, 110 leaves, 3 steps):

$$\frac{1}{2 a \sqrt{a + \frac{b}{x^4}} x} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right] - \frac{4 a^{5/4} b^{1/4} \sqrt{a + \frac{b}{x^4}}}{2 a \sqrt{a + \frac{b}{x^4}} x}$$

Result (type 4, 105 leaves):

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x + i \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x \right], -1 \right]}{2 a \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} \sqrt{a + \frac{b}{x^4}} x^2}$$

Problem 2095: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{3/2} x^4} dx$$

Optimal (type 4, 241 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{1}{2 a \sqrt{a + \frac{b}{x^4}} x^3} + \frac{\sqrt{a + \frac{b}{x^4}}}{2 a \sqrt{b} \left(\sqrt{a + \frac{b}{x^2}}\right) x} - \frac{\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} b^{3/4} \sqrt{a + \frac{b}{x^4}}} + \\
 & \frac{\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{3/4} b^{3/4} \sqrt{a + \frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 166 leaves):

$$\begin{aligned}
 & \left(i \left(\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x^3 - \sqrt{b} \sqrt{1 + \frac{a x^4}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] + \right. \right. \\
 & \left. \left. \sqrt{b} \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right) / \left(2 \left(\frac{i \sqrt{a}}{\sqrt{b}} \right)^{3/2} b^{3/2} \sqrt{a + \frac{b}{x^4}} x^2 \right)
 \end{aligned}$$

Problem 2100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(a + \frac{b}{x^4}\right)^{5/2}} dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{x^3}{6 a \left(a + \frac{b}{x^4}\right)^{3/2}} - \frac{3 x^3}{4 a^2 \sqrt{a + \frac{b}{x^4}}} + \frac{5 \sqrt{a + \frac{b}{x^4}} x^3}{4 a^3} + \\
 & \frac{5 b^{3/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a + \frac{b}{x^2}}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{8 a^{13/4} \sqrt{a + \frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 118 leaves):

$$\frac{\frac{15 b^2 x + 21 a b x^5 + 4 a^2 x^9}{b + a x^4} + \frac{15 i b \sqrt{1 + \frac{a x^4}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}}{12 a^3 \sqrt{a + \frac{b}{x^4}} x^2}$$

Problem 2101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{5/2}} dx$$

Optimal (type 4, 277 leaves, 7 steps):

$$\begin{aligned} & -\frac{7 \sqrt{b} \sqrt{a + \frac{b}{x^4}}}{4 a^3 \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) x} - \frac{x}{6 a \left(a + \frac{b}{x^4}\right)^{3/2}} - \frac{7 x}{12 a^2 \sqrt{a + \frac{b}{x^4}}} + \frac{7 \sqrt{a + \frac{b}{x^4}} x}{4 a^3} + \\ & \frac{7 b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \operatorname{EllipticE}\left[2 \operatorname{ArcCot}\left[\frac{a^{3/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{11/4} \sqrt{a + \frac{b}{x^4}}} - \\ & \frac{7 b^{1/4} \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcCot}\left[\frac{a^{3/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{8 a^{11/4} \sqrt{a + \frac{b}{x^4}}} \end{aligned}$$

Result (type 4, 153 leaves):

$$\begin{aligned} & \frac{1}{4 \left(a + \frac{b}{x^4}\right)^{5/2} x^{10}} (b + a x^4)^2 \left(-\frac{x^3 (7 b + 9 a x^4)}{3 a^2 (b + a x^4)} + \frac{1}{a^2 \left(\frac{i \sqrt{a}}{\sqrt{b}}\right)^{3/2}} 7 i \sqrt{1 + \frac{a x^4}{b}} \right. \\ & \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right) \end{aligned}$$

Problem 2102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{5/2} x^2} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{1}{6 a \left(a + \frac{b}{x^4}\right)^{3/2} x} - \frac{5}{12 a^2 \sqrt{a + \frac{b}{x^4}} x} - \frac{5 \sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{24 a^{9/4} b^{1/4} \sqrt{a + \frac{b}{x^4}}}$$

Result (type 4, 107 leaves):

$$\frac{-\frac{5 b x + 7 a x^5}{b + a x^4} - \frac{5 i \sqrt{1 + \frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}}{12 a^2 \sqrt{a + \frac{b}{x^4}} x^2}$$

Problem 2103: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{x^4}\right)^{5/2} x^4} dx$$

Optimal (type 4, 262 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{1}{6 a \left(a + \frac{b}{x^4}\right)^{3/2} x^3} - \frac{1}{4 a^2 \sqrt{a + \frac{b}{x^4}} x^3} + \frac{\sqrt{a + \frac{b}{x^4}}}{4 a^2 \sqrt{b} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) x} \\
 & \frac{\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{7/4} b^{3/4} \sqrt{a + \frac{b}{x^4}}} + \\
 & \frac{\sqrt{\frac{a + \frac{b}{x^4}}{\left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a} + \frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{8 a^{7/4} b^{3/4} \sqrt{a + \frac{b}{x^4}}}
 \end{aligned}$$

Result (type 4, 155 leaves):

$$\begin{aligned}
 & \frac{1}{4 \left(a + \frac{b}{x^4}\right)^{5/2} x^{10}} (b + a x^4)^2 \left(\frac{b x^3 + 3 a x^7}{3 a b^2 + 3 a^2 b x^4} + \frac{1}{a^2} i \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{1 + \frac{a x^4}{b}} \right. \\
 & \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right] \right) \right)
 \end{aligned}$$

Problem 2105: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^5}} x} dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a + \frac{b}{x^5}}}{\sqrt{a}}\right]}{5 \sqrt{a}}$$

Result (type 8, 17 leaves):

$$\int \frac{1}{\sqrt{a + \frac{b}{x^5}} x} dx$$

Problem 2106: Unable to integrate problem.

$$\int \frac{1}{\sqrt{-a + \frac{b}{x^5} x}} dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-a + \frac{b}{x^5}}}{\sqrt{a}}\right]}{5 \sqrt{a}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{\sqrt{-a + \frac{b}{x^5} x}} dx$$

Problem 2147: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sqrt{x})^5}{x^4} dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{(a + b \sqrt{x})^6}{3 a x^3}$$

Result (type 2, 63 leaves):

$$-\frac{a^5 + 6 a^4 b \sqrt{x} + 15 a^3 b^2 x + 20 a^2 b^3 x^{3/2} + 15 a b^4 x^2 + 6 b^5 x^{5/2}}{3 x^3}$$

Problem 2155: Result more than twice size of optimal antiderivative.

$$\int (a + b \sqrt{x})^{10} dx$$

Optimal (type 2, 38 leaves, 3 steps):

$$-\frac{2 a (a + b \sqrt{x})^{11}}{11 b^2} + \frac{(a + b \sqrt{x})^{12}}{6 b^2}$$

Result (type 2, 131 leaves):

$$a^{10} x + \frac{20}{3} a^9 b x^{3/2} + \frac{45}{2} a^8 b^2 x^2 + 48 a^7 b^3 x^{5/2} + 70 a^6 b^4 x^3 + 72 a^5 b^5 x^{7/2} + \frac{105}{2} a^4 b^6 x^4 + \frac{80}{3} a^3 b^7 x^{9/2} + 9 a^2 b^8 x^5 + \frac{20}{11} a b^9 x^{11/2} + \frac{b^{10} x^6}{6}$$

Problem 2162: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sqrt{x})^{10}}{x^7} dx$$

Optimal (type 2, 46 leaves, 3 steps):

$$-\frac{(a + b \sqrt{x})^{11}}{6 a x^6} + \frac{b (a + b \sqrt{x})^{11}}{66 a^2 x^{11/2}}$$

Result (type 2, 124 leaves):

$$-\frac{1}{66 x^6} \left(11 a^{10} + 120 a^9 b \sqrt{x} + 594 a^8 b^2 x + 1760 a^7 b^3 x^{3/2} + 3465 a^6 b^4 x^2 + 4752 a^5 b^5 x^{5/2} + 4620 a^4 b^6 x^3 + 3168 a^3 b^7 x^{7/2} + 1485 a^2 b^8 x^4 + 440 a b^9 x^{9/2} + 66 b^{10} x^5 \right)$$

Problem 2171: Result more than twice size of optimal antiderivative.

$$\int (a + b \sqrt{x})^{15} x dx$$

Optimal (type 2, 80 leaves, 3 steps):

$$-\frac{a^3 (a + b \sqrt{x})^{16}}{8 b^4} + \frac{6 a^2 (a + b \sqrt{x})^{17}}{17 b^4} - \frac{a (a + b \sqrt{x})^{18}}{3 b^4} + \frac{2 (a + b \sqrt{x})^{19}}{19 b^4}$$

Result (type 2, 199 leaves):

$$\frac{a^{15} x^2}{2} + 6 a^{14} b x^{5/2} + 35 a^{13} b^2 x^3 + 130 a^{12} b^3 x^{7/2} + \frac{1365}{4} a^{11} b^4 x^4 + \frac{2002}{3} a^{10} b^5 x^{9/2} + 1001 a^9 b^6 x^5 + 1170 a^8 b^7 x^{11/2} + \frac{2145}{2} a^7 b^8 x^6 + 770 a^6 b^9 x^{13/2} + 429 a^5 b^{10} x^7 + 182 a^4 b^{11} x^{15/2} + \frac{455}{8} a^3 b^{12} x^8 + \frac{210}{17} a^2 b^{13} x^{17/2} + \frac{5}{3} a b^{14} x^9 + \frac{2}{19} b^{15} x^{19/2}$$

Problem 2172: Result more than twice size of optimal antiderivative.

$$\int (a + b \sqrt{x})^{15} dx$$

Optimal (type 2, 38 leaves, 3 steps):

$$-\frac{a (a + b \sqrt{x})^{16}}{8 b^2} + \frac{2 (a + b \sqrt{x})^{17}}{17 b^2}$$

Result (type 2, 190 leaves):

$$a^{15} x + 10 a^{14} b x^{3/2} + \frac{105}{2} a^{13} b^2 x^2 + 182 a^{12} b^3 x^{5/2} + 455 a^{11} b^4 x^3 +$$

$$858 a^{10} b^5 x^{7/2} + \frac{5005}{4} a^9 b^6 x^4 + 1430 a^8 b^7 x^{9/2} + 1287 a^7 b^8 x^5 + 910 a^6 b^9 x^{11/2} +$$

$$\frac{1001}{2} a^5 b^{10} x^6 + 210 a^4 b^{11} x^{13/2} + 65 a^3 b^{12} x^7 + 14 a^2 b^{13} x^{15/2} + \frac{15}{8} a b^{14} x^8 + \frac{2}{17} b^{15} x^{17/2}$$

Problem 2180: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sqrt{x})^{15}}{x^9} dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{(a + b \sqrt{x})^{16}}{8 a x^8}$$

Result (type 2, 183 leaves):

$$-\frac{1}{8 x^8} \left(a^{15} + 16 a^{14} b \sqrt{x} + 120 a^{13} b^2 x + 560 a^{12} b^3 x^{3/2} + 1820 a^{11} b^4 x^2 + 4368 a^{10} b^5 x^{5/2} + \right.$$

$$8008 a^9 b^6 x^3 + 11440 a^8 b^7 x^{7/2} + 12870 a^7 b^8 x^4 + 11440 a^6 b^9 x^{9/2} + 8008 a^5 b^{10} x^5 +$$

$$\left. 4368 a^4 b^{11} x^{11/2} + 1820 a^3 b^{12} x^6 + 560 a^2 b^{13} x^{13/2} + 120 a b^{14} x^7 + 16 b^{15} x^{15/2} \right)$$

Problem 2181: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sqrt{x})^{15}}{x^{10}} dx$$

Optimal (type 2, 70 leaves, 4 steps):

$$-\frac{(a + b \sqrt{x})^{16}}{9 a x^9} + \frac{2 b (a + b \sqrt{x})^{16}}{153 a^2 x^{17/2}} - \frac{b^2 (a + b \sqrt{x})^{16}}{1224 a^3 x^8}$$

Result (type 2, 185 leaves):

$$-\frac{1}{1224 x^9}$$

$$\left(136 a^{15} + 2160 a^{14} b \sqrt{x} + 16065 a^{13} b^2 x + 74256 a^{12} b^3 x^{3/2} + 238680 a^{11} b^4 x^2 + 565488 a^{10} b^5 x^{5/2} + \right.$$

$$1021020 a^9 b^6 x^3 + 1432080 a^8 b^7 x^{7/2} + 1575288 a^7 b^8 x^4 + 1361360 a^6 b^9 x^{9/2} + 918918 a^5 b^{10} x^5 +$$

$$\left. 477360 a^4 b^{11} x^{11/2} + 185640 a^3 b^{12} x^6 + 51408 a^2 b^{13} x^{13/2} + 9180 a b^{14} x^7 + 816 b^{15} x^{15/2} \right)$$

Problem 2216: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + b \sqrt{x})^5} dx$$

Optimal (type 2, 21 leaves, 1 step):

$$\frac{x^2}{2 a (a + b \sqrt{x})^4}$$

Result (type 2, 50 leaves):

$$\frac{a^3 + 4 a^2 b \sqrt{x} + 6 a b^2 x + 4 b^3 x^{3/2}}{2 b^4 (a + b \sqrt{x})^4}$$

Problem 2268: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{(a + b x^{3/2})^{2/3}} dx$$

Optimal (type 5, 42 leaves, 3 steps):

$$\frac{x^5 (a + b x^{3/2})^{1/3} \operatorname{Hypergeometric2F1}\left[1, \frac{11}{3}, \frac{13}{3}, -\frac{b x^{3/2}}{a}\right]}{5 a}$$

Result (type 5, 103 leaves):

$$\left(\sqrt{x} \left(14 a^3 + 7 a^2 b x^{3/2} - 2 a b^2 x^3 + 5 b^3 x^{9/2} - 14 a^3 \left(1 + \frac{b x^{3/2}}{a} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^{3/2}}{a}\right] \right) \right) / \left(20 b^3 (a + b x^{3/2})^{2/3} \right)$$

Problem 2269: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(a + b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 198 leaves, 10 steps):

$$\begin{aligned} & -\frac{5 a x (a + b x^{3/2})^{1/3}}{9 b^2} + \frac{x^{5/2} (a + b x^{3/2})^{1/3}}{3 b} - \frac{10 a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}} \\ & + \frac{10 a^2 \operatorname{Log}\left[1 - \frac{b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}\right]}{27 b^{8/3}} + \frac{5 a^2 \operatorname{Log}\left[1 + \frac{b^{2/3} x}{(a + b x^{3/2})^{2/3}} + \frac{b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}\right]}{27 b^{8/3}} \end{aligned}$$

Result (type 5, 87 leaves):

$$\left(-5 a^2 x - 2 a b x^{5/2} + 3 b^2 x^4 + 5 a^2 x \left(1 + \frac{b x^{3/2}}{a} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^{3/2}}{a}\right] \right) / \left(9 b^2 (a + b x^{3/2})^{2/3} \right)$$

Problem 2272: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 140 leaves, 8 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{2 \operatorname{Log}\left[1 - \frac{b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}\right]}{3 b^{2/3}} + \frac{\operatorname{Log}\left[1 + \frac{b^{2/3} x}{(a + b x^{3/2})^{2/3}} + \frac{b^{1/3} \sqrt{x}}{(a + b x^{3/2})^{1/3}}\right]}{3 b^{2/3}}$$

Result (type 5, 53 leaves):

$$\frac{x \left(\frac{a + b x^{3/2}}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^{3/2}}{a}\right]}{(a + b x^{3/2})^{2/3}}$$

Problem 2273: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x^{3/2})^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3}} - \frac{\operatorname{Log}[x]}{2 a^{2/3}} + \frac{\operatorname{Log}\left[a^{1/3} - (a + b x^{3/2})^{1/3}\right]}{a^{2/3}}$$

Result (type 5, 52 leaves):

$$-\frac{\left(1 + \frac{a}{b x^{3/2}}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^{3/2}}\right]}{(a + b x^{3/2})^{2/3}}$$

Problem 2276: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a + b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 148 leaves, 7 steps):

$$-\frac{(a + b x^{3/2})^{1/3}}{3 a x^3} + \frac{5 b (a + b x^{3/2})^{1/3}}{9 a^2 x^{3/2}} - \frac{10 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x^{3/2})^{1/3}}{\sqrt{3} a^{1/3}}\right]}{9 \sqrt{3} a^{8/3}} - \frac{5 b^2 \operatorname{Log}[x]}{18 a^{8/3}} + \frac{5 b^2 \operatorname{Log}\left[a^{1/3} - (a + b x^{3/2})^{1/3}\right]}{9 a^{8/3}}$$

Result (type 5, 91 leaves):

$$\frac{\left(-3 a^2 + 2 a b x^{3/2} + 5 b^2 x^3 - 5 b^2 \left(1 + \frac{a}{b x^{3/2}}\right)^{2/3} x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x^{3/2}}\right]\right)}{\left(9 a^2 x^3 (a + b x^{3/2})^{2/3}\right)}$$

Problem 2312: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^{1/3})^5}{x^3} dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{(a + b x^{1/3})^6}{2 a x^2}$$

Result (type 2, 65 leaves):

$$-\frac{a^5 + 6 a^4 b x^{1/3} + 15 a^3 b^2 x^{2/3} + 20 a^2 b^3 x + 15 a b^4 x^{4/3} + 6 b^5 x^{5/3}}{2 x^2}$$

Problem 2321: Result more than twice size of optimal antiderivative.

$$\int (a + b x^{1/3})^{10} dx$$

Optimal (type 2, 59 leaves, 3 steps):

$$\frac{3 a^2 (a + b x^{1/3})^{11}}{11 b^3} - \frac{a (a + b x^{1/3})^{12}}{2 b^3} + \frac{3 (a + b x^{1/3})^{13}}{13 b^3}$$

Result (type 2, 133 leaves):

$$a^{10} x + \frac{15}{2} a^9 b x^{4/3} + 27 a^8 b^2 x^{5/3} + 60 a^7 b^3 x^2 + 90 a^6 b^4 x^{7/3} + \frac{189}{2} a^5 b^5 x^{8/3} + 70 a^4 b^6 x^3 + 36 a^3 b^7 x^{10/3} + \frac{135}{11} a^2 b^8 x^{11/3} + \frac{5}{2} a b^9 x^4 + \frac{3}{13} b^{10} x^{13/3}$$

Problem 2326: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^{1/3})^{10}}{x^5} dx$$

Optimal (type 2, 46 leaves, 3 steps):

$$-\frac{(a + b x^{1/3})^{11}}{4 a x^4} + \frac{b (a + b x^{1/3})^{11}}{44 a^2 x^{11/3}}$$

Result (type 2, 128 leaves):

$$-\frac{1}{44 x^4} \left(11 a^{10} + 120 a^9 b x^{1/3} + 594 a^8 b^2 x^{2/3} + 1760 a^7 b^3 x + 3465 a^6 b^4 x^{4/3} + 4752 a^5 b^5 x^{5/3} + 4620 a^4 b^6 x^2 + 3168 a^3 b^7 x^{7/3} + 1485 a^2 b^8 x^{8/3} + 440 a b^9 x^3 + 66 b^{10} x^{10/3}\right)$$

Problem 2337: Result more than twice size of optimal antiderivative.

$$\int (a + b x^{1/3})^{15} dx$$

Optimal (type 2, 59 leaves, 3 steps):

$$\frac{3 a^2 (a + b x^{1/3})^{16}}{16 b^3} - \frac{6 a (a + b x^{1/3})^{17}}{17 b^3} + \frac{(a + b x^{1/3})^{18}}{6 b^3}$$

Result (type 2, 204 leaves):

$$\begin{aligned} & a^{15} x + \frac{45}{4} a^{14} b x^{4/3} + 63 a^{13} b^2 x^{5/3} + \frac{455}{2} a^{12} b^3 x^2 + 585 a^{11} b^4 x^{7/3} + \\ & \frac{9009}{8} a^{10} b^5 x^{8/3} + \frac{5005}{3} a^9 b^6 x^3 + \frac{3861}{2} a^8 b^7 x^{10/3} + 1755 a^7 b^8 x^{11/3} + \frac{5005}{4} a^6 b^9 x^4 + \\ & 693 a^5 b^{10} x^{13/3} + \frac{585}{2} a^4 b^{11} x^{14/3} + 91 a^3 b^{12} x^5 + \frac{315}{16} a^2 b^{13} x^{16/3} + \frac{45}{17} a b^{14} x^{17/3} + \frac{b^{15} x^6}{6} \end{aligned}$$

Problem 2343: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^{1/3})^{15}}{x^7} dx$$

Optimal (type 2, 72 leaves, 4 steps):

$$-\frac{(a + b x^{1/3})^{16}}{6 a x^6} + \frac{b (a + b x^{1/3})^{16}}{51 a^2 x^{17/3}} - \frac{b^2 (a + b x^{1/3})^{16}}{816 a^3 x^{16/3}}$$

Result (type 2, 189 leaves):

$$\begin{aligned} & -\frac{1}{816 x^6} \\ & (136 a^{15} + 2160 a^{14} b x^{1/3} + 16065 a^{13} b^2 x^{2/3} + 74256 a^{12} b^3 x + 238680 a^{11} b^4 x^{4/3} + 565488 a^{10} b^5 x^{5/3} + \\ & 1021020 a^9 b^6 x^2 + 1432080 a^8 b^7 x^{7/3} + 1575288 a^7 b^8 x^{8/3} + 1361360 a^6 b^9 x^3 + 918918 a^5 b^{10} x^{10/3} + \\ & 477360 a^4 b^{11} x^{11/3} + 185640 a^3 b^{12} x^4 + 51408 a^2 b^{13} x^{13/3} + 9180 a b^{14} x^{14/3} + 816 b^{15} x^5) \end{aligned}$$

Problem 2386: Result unnecessarily involves higher level functions.

$$\int \left(a + \frac{b}{x^{3/2}}\right)^{2/3} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$\left(a + \frac{b}{x^{3/2}}\right)^{2/3} x - \frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3}}{\left(a + \frac{b}{x^{3/2}}\right)^{1/3} \sqrt{x}}}{\sqrt{3}}\right]}{\sqrt{3}} + b^{2/3} \operatorname{Log}\left[\left(a + \frac{b}{x^{3/2}}\right)^{1/3} - \frac{b^{1/3}}{\sqrt{x}}\right]$$

Result (type 5, 53 leaves):

$$\frac{\left(a + \frac{b}{x^{3/2}}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{b}{a x^{3/2}}\right]}{\left(\frac{a + \frac{b}{x^{3/2}}}{a}\right)^{2/3}}$$

Problem 2471: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^n)^2} dx$$

Optimal (type 5, 24 leaves, 1 step):

$$\frac{x \operatorname{Hypergeometric2F1}\left[2, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^2}$$

Result (type 5, 49 leaves):

$$\frac{x \left(a + (-1 + n) (a + b x^n)\right) \operatorname{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^2 n (a + b x^n)}$$

Problem 2475: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + b x^n)^3} dx$$

Optimal (type 5, 33 leaves, 1 step):

$$\frac{x^2 \operatorname{Hypergeometric2F1}\left[3, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]}{2 a^3}$$

Result (type 5, 74 leaves):

$$\frac{1}{2 a^3 n^2} x^2 \left(\frac{a (a (-2 + 3 n) + 2 b (-1 + n) x^n)}{(a + b x^n)^2} + (2 - 3 n + n^2) \operatorname{Hypergeometric2F1}\left[1, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right] \right)$$

Problem 2476: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^n)^3} dx$$

Optimal (type 5, 24 leaves, 1 step):

$$\frac{x \operatorname{Hypergeometric2F1}\left[3, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^3}$$

Result (type 5, 71 leaves):

$$\frac{1}{2 a^3 n^2} x \left(\frac{a (a (-1 + 3 n) + b (-1 + 2 n) x^n)}{(a + b x^n)^2} + (1 - 3 n + 2 n^2) \operatorname{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right] \right)$$

Problem 2478: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 (a + b x^n)^3} dx$$

Optimal (type 5, 34 leaves, 1 step):

$$\frac{\text{Hypergeometric2F1}\left[3, -\frac{1}{n}, -\frac{1-n}{n}, -\frac{b x^n}{a}\right]}{a^3 x}$$

Result (type 5, 76 leaves):

$$\frac{1}{2 a^3 n^2 x} \left(\frac{a (a + 3 a n + b (1 + 2 n) x^n)}{(a + b x^n)^2} - (1 + 3 n + 2 n^2) \text{Hypergeometric2F1}\left[1, -\frac{1}{n}, \frac{-1+n}{n}, -\frac{b x^n}{a}\right] \right)$$

Problem 2479: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a + b x^n)^3} dx$$

Optimal (type 5, 36 leaves, 1 step):

$$\frac{\text{Hypergeometric2F1}\left[3, -\frac{2}{n}, -\frac{2-n}{n}, -\frac{b x^n}{a}\right]}{2 a^3 x^2}$$

Result (type 5, 75 leaves):

$$\frac{1}{2 a^3 n^2 x^2} \left(\frac{a (a (2 + 3 n) + 2 b (1 + n) x^n)}{(a + b x^n)^2} - (2 + 3 n + n^2) \text{Hypergeometric2F1}\left[1, -\frac{2}{n}, \frac{-2+n}{n}, -\frac{b x^n}{a}\right] \right)$$

Problem 2485: Result more than twice size of optimal antiderivative.

$$\int x (a + b x^n)^{3/2} dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2 (a + b x^n)^{5/2} \text{Hypergeometric2F1}\left[1, \frac{5}{2} + \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]}{2 a}$$

Result (type 5, 102 leaves):

$$\left(x^2 \left(4 (a + b x^n) (4 a (1 + n) + b (4 + n) x^n) + \right. \right. \\ \left. \left. 3 a^2 n^2 \sqrt{1 + \frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right] \right) \right) / \left(2 (4 + n) (4 + 3 n) \sqrt{a + b x^n} \right)$$

Problem 2486: Result more than twice size of optimal antiderivative.

$$\int (a + b x^n)^{3/2} dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{x (a + b x^n)^{5/2} \operatorname{Hypergeometric2F1}\left[1, \frac{5}{2} + \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a}$$

Result (type 5, 94 leaves):

$$\left(x \left(2 (a + b x^n) (a (2 + 4 n) + b (2 + n) x^n) + 3 a^2 n^2 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right] \right) \right) / \left((2 + n) (2 + 3 n) \sqrt{a + b x^n} \right)$$

Problem 2488: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^n)^{3/2}}{x^2} dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{(a + b x^n)^{5/2} \operatorname{Hypergeometric2F1}\left[1, \frac{5}{2} - \frac{1}{n}, -\frac{1-n}{n}, -\frac{b x^n}{a}\right]}{a x}$$

Result (type 5, 100 leaves):

$$\left(2 (a + b x^n) (a (-2 + 4 n) + b (-2 + n) x^n) - 3 a^2 n^2 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{n}, \frac{-1+n}{n}, -\frac{b x^n}{a}\right] \right) / \left((-2 + n) (-2 + 3 n) x \sqrt{a + b x^n} \right)$$

Problem 2490: Result more than twice size of optimal antiderivative.

$$\int x (a + b x^n)^{5/2} dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2 (a + b x^n)^{7/2} \operatorname{Hypergeometric2F1}\left[1, \frac{7}{2} + \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]}{2 a}$$

Result (type 5, 144 leaves):

$$\left(x^2 \left(4 (a + b x^n) (a^2 (16 + 36 n + 23 n^2) + a b (32 + 52 n + 11 n^2) x^n + b^2 (16 + 16 n + 3 n^2) x^{2n}) + 15 a^3 n^3 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a} \right] \right) \right) / \left(2 (4+n) (4+3n) (4+5n) \sqrt{a + b x^n} \right)$$

Problem 2491: Result more than twice size of optimal antiderivative.

$$\int (a + b x^n)^{5/2} dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{x (a + b x^n)^{7/2} \operatorname{Hypergeometric2F1} \left[1, \frac{7}{2} + \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a} \right]}{a}$$

Result (type 5, 135 leaves):

$$\left(x \left(2 (a + b x^n) (a^2 (4 + 18 n + 23 n^2) + a b (8 + 26 n + 11 n^2) x^n + b^2 (4 + 8 n + 3 n^2) x^{2n}) + 15 a^3 n^3 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a} \right] \right) \right) / \left((2+n) (2+3n) (2+5n) \sqrt{a + b x^n} \right)$$

Problem 2493: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^n)^{5/2}}{x^2} dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{(a + b x^n)^{7/2} \operatorname{Hypergeometric2F1} \left[1, \frac{7}{2} - \frac{1}{n}, -\frac{1-n}{n}, -\frac{b x^n}{a} \right]}{a x}$$

Result (type 5, 141 leaves):

$$\left(2 (a + b x^n) (a^2 (4 - 18 n + 23 n^2) + a b (8 - 26 n + 11 n^2) x^n + b^2 (4 - 8 n + 3 n^2) x^{2n}) - \right. \\ \left. 15 a^3 n^3 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{n}, \frac{-1+n}{n}, -\frac{b x^n}{a}\right] \right) / \\ \left((-2+n) (-2+3n) (-2+5n) x \sqrt{a + b x^n} \right)$$

Problem 2494: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^n)^{5/2}}{x^3} dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{(a + b x^n)^{7/2} \operatorname{Hypergeometric2F1}\left[1, \frac{7}{2} - \frac{2}{n}, -\frac{2-n}{n}, -\frac{b x^n}{a}\right]}{2 a x^2}$$

Result (type 5, 144 leaves):

$$\left(4 (a + b x^n) (a^2 (16 - 36 n + 23 n^2) + a b (32 - 52 n + 11 n^2) x^n + b^2 (16 - 16 n + 3 n^2) x^{2n}) - \right. \\ \left. 15 a^3 n^3 \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2}{n}, \frac{-2+n}{n}, -\frac{b x^n}{a}\right] \right) / \\ \left(2 (-4+n) (-4+3n) (-4+5n) x^2 \sqrt{a + b x^n} \right)$$

Problem 2505: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + b x^n)^{5/2}} dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2 \operatorname{Hypergeometric2F1}\left[1, -\frac{3}{2} + \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]}{2 a (a + b x^n)^{3/2}}$$

Result (type 5, 100 leaves):

$$\left(x^2 \left(4 a n + 4 (-4 + 3 n) (a + b x^n) + (16 - 16 n + 3 n^2) (a + b x^n) \right. \right. \\ \left. \left. \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a} \right] \right) \right) / \left(6 a^2 n^2 (a + b x^n)^{3/2} \right)$$

Problem 2506: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^n)^{5/2}} dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, -\frac{3}{2} + \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a} \right]}{a (a + b x^n)^{3/2}}$$

Result (type 5, 94 leaves):

$$\left(x \left(2 a n + 2 (-2 + 3 n) (a + b x^n) + (4 - 8 n + 3 n^2) (a + b x^n) \right. \right. \\ \left. \left. \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a} \right] \right) \right) / \left(3 a^2 n^2 (a + b x^n)^{3/2} \right)$$

Problem 2508: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 (a + b x^n)^{5/2}} dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, -\frac{3}{2} - \frac{1}{n}, -\frac{1-n}{n}, -\frac{b x^n}{a} \right]}{a x (a + b x^n)^{3/2}}$$

Result (type 5, 101 leaves):

$$\left(2 a n + 2 (2 + 3 n) (a + b x^n) - (4 + 8 n + 3 n^2) (a + b x^n) \sqrt{1 + \frac{b x^n}{a}} \right. \\ \left. \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{1}{n}, \frac{-1+n}{n}, -\frac{b x^n}{a} \right] \right) / \left(3 a^2 n^2 x (a + b x^n)^{3/2} \right)$$

Problem 2510: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^n)^{1/3}}{x} dx$$

Optimal (type 3, 106 leaves, 6 steps):

$$\frac{3 (a + b x^n)^{1/3}}{n} - \frac{\sqrt{3} a^{1/3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x^n)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{n} - \frac{1}{2} a^{1/3} \operatorname{Log}[x] + \frac{3 a^{1/3} \operatorname{Log}\left[a^{1/3} - (a + b x^n)^{1/3}\right]}{2 n}$$

Result (type 5, 68 leaves):

$$\frac{6 (a + b x^n) - 3 a \left(1 + \frac{a x^{-n}}{b}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a x^{-n}}{b}\right]}{2 n (a + b x^n)^{2/3}}$$

Problem 2554: Result more than twice size of optimal antiderivative.

$$\int x^{-1-6n} (a + b x^n)^5 dx$$

Optimal (type 3, 24 leaves, 1 step):

$$-\frac{x^{-6n} (a + b x^n)^6}{6 a n}$$

Result (type 3, 72 leaves):

$$-\frac{x^{-6n} (a^5 + 6 a^4 b x^n + 15 a^3 b^2 x^{2n} + 20 a^2 b^3 x^{3n} + 15 a b^4 x^{4n} + 6 b^5 x^{5n})}{6 n}$$

Problem 2566: Result more than twice size of optimal antiderivative.

$$\int x^{-1+2n} (a + b x^n)^8 dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$-\frac{a (a + b x^n)^9}{9 b^2 n} + \frac{(a + b x^n)^{10}}{10 b^2 n}$$

Result (type 3, 113 leaves):

$$\frac{1}{90 n} x^{2n} (45 a^8 + 240 a^7 b x^n + 630 a^6 b^2 x^{2n} + 1008 a^5 b^3 x^{3n} + 1050 a^4 b^4 x^{4n} + 720 a^3 b^5 x^{5n} + 315 a^2 b^6 x^{6n} + 80 a b^7 x^{7n} + 9 b^8 x^{8n})$$

Problem 2577: Result more than twice size of optimal antiderivative.

$$\int x^{-1-9n} (a + b x^n)^8 dx$$

Optimal (type 3, 24 leaves, 1 step):

$$-\frac{x^{-9n} (a + b x^n)^9}{9 a n}$$

Result (type 3, 111 leaves):

$$-\frac{1}{9n} x^{-9n} (a^8 + 9 a^7 b x^n + 36 a^6 b^2 x^{2n} + 84 a^5 b^3 x^{3n} + 126 a^4 b^4 x^{4n} + 126 a^3 b^5 x^{5n} + 84 a^2 b^6 x^{6n} + 36 a b^7 x^{7n} + 9 b^8 x^{8n})$$

Problem 2578: Result more than twice size of optimal antiderivative.

$$\int x^{-1-10n} (a + b x^n)^8 dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$-\frac{x^{-10n} (a + b x^n)^9}{10 a n} + \frac{b x^{-9n} (a + b x^n)^9}{90 a^2 n}$$

Result (type 3, 113 leaves):

$$-\frac{1}{90n} x^{-10n} (9 a^8 + 80 a^7 b x^n + 315 a^6 b^2 x^{2n} + 720 a^5 b^3 x^{3n} + 1050 a^4 b^4 x^{4n} + 1008 a^3 b^5 x^{5n} + 630 a^2 b^6 x^{6n} + 240 a b^7 x^{7n} + 45 b^8 x^{8n})$$

Problem 2585: Result more than twice size of optimal antiderivative.

$$\int x^{12} (a + b x^{13})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{13})^{13}}{169 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{13}}{13} + \frac{6}{13} a^{11} b x^{26} + \frac{22}{13} a^{10} b^2 x^{39} + \frac{55}{13} a^9 b^3 x^{52} + \frac{99}{13} a^8 b^4 x^{65} + \frac{132}{13} a^7 b^5 x^{78} + \frac{132}{13} a^6 b^6 x^{91} + \frac{99}{13} a^5 b^7 x^{104} + \frac{55}{13} a^4 b^8 x^{117} + \frac{22}{13} a^3 b^9 x^{130} + \frac{6}{13} a^2 b^{10} x^{143} + \frac{1}{13} a b^{11} x^{156} + \frac{b^{12} x^{169}}{169}$$

Problem 2586: Result more than twice size of optimal antiderivative.

$$\int x^{24} (a + b x^{25})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{25})^{13}}{325 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{25}}{25} + \frac{6}{25} a^{11} b x^{50} + \frac{22}{25} a^{10} b^2 x^{75} + \frac{11}{5} a^9 b^3 x^{100} + \frac{99}{25} a^8 b^4 x^{125} + \frac{132}{25} a^7 b^5 x^{150} + \frac{132}{25} a^6 b^6 x^{175} + \frac{99}{25} a^5 b^7 x^{200} + \frac{11}{5} a^4 b^8 x^{225} + \frac{22}{25} a^3 b^9 x^{250} + \frac{6}{25} a^2 b^{10} x^{275} + \frac{1}{25} a b^{11} x^{300} + \frac{b^{12} x^{325}}{325}$$

Problem 2587: Result more than twice size of optimal antiderivative.

$$\int x^{36} (a + b x^{37})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} + \frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481}$$

Problem 2633: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{2n}{3}}}{a + b x^n} dx$$

Optimal (type 3, 160 leaves, 8 steps):

$$-\frac{3 x^{-2n/3}}{2 a n} + \frac{\sqrt{3} b^{2/3} \text{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} x^{n/3}}{\sqrt{3} a^{1/3}}\right]}{a^{5/3} n} - \frac{b^{2/3} \text{Log}\left[a^{1/3} + b^{1/3} x^{n/3}\right]}{a^{5/3} n} + \frac{b^{2/3} \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} x^{n/3} + b^{2/3} x^{2n/3}\right]}{2 a^{5/3} n}$$

Result (type 7, 60 leaves):

$$\frac{-9 a x^{-2n/3} + 2 b \text{RootSum}\left[b + a \sqrt[3]{1}, \frac{n \text{Log}[x] + 3 \text{Log}\left[x^{-n/3} - \sqrt[3]{1}\right]}{\sqrt[3]{1}}\right]}{6 a^2 n}$$

Problem 2634: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{3n}{4}}}{a + b x^n} dx$$

Optimal (type 3, 236 leaves, 11 steps):

$$-\frac{4 x^{-3 n/4}}{3 a n} + \frac{\sqrt{2} b^{3/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} x^{n/4}}{a^{1/4}}\right]}{a^{7/4} n} - \frac{\sqrt{2} b^{3/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x^{n/4}}{a^{1/4}}\right]}{a^{7/4} n} +$$

$$\frac{b^{3/4} \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x^{n/4} + \sqrt{b} x^{n/2}\right]}{\sqrt{2} a^{7/4} n} - \frac{b^{3/4} \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x^{n/4} + \sqrt{b} x^{n/2}\right]}{\sqrt{2} a^{7/4} n}$$

Result (type 7, 60 leaves):

$$\frac{-16 a x^{-3 n/4} + 3 b \operatorname{RootSum}\left[b + a \#1^4 \&, \frac{n \operatorname{Log}[x] + 4 \operatorname{Log}\left[x^{-n/4} - \#1\right]}{\#1}\right] \&]}{12 a^2 n}$$

Problem 2637: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{3}}}{a + b x^n} dx$$

Optimal (type 3, 158 leaves, 9 steps):

$$-\frac{3 x^{-n/3}}{a n} - \frac{\sqrt{3} b^{1/3} \operatorname{ArcTan}\left[\frac{b^{1/3} - 2 a^{1/3} x^{-n/3}}{\sqrt{3} b^{1/3}}\right]}{a^{4/3} n} +$$

$$\frac{b^{1/3} \operatorname{Log}\left[b^{1/3} + a^{1/3} x^{-n/3}\right]}{a^{4/3} n} - \frac{b^{1/3} \operatorname{Log}\left[b^{2/3} + a^{2/3} x^{-2 n/3} - a^{1/3} b^{1/3} x^{-n/3}\right]}{2 a^{4/3} n}$$

Result (type 7, 59 leaves):

$$\frac{-9 a x^{-n/3} + b \operatorname{RootSum}\left[b + a \#1^3 \&, \frac{n \operatorname{Log}[x] + 3 \operatorname{Log}\left[x^{-n/3} - \#1\right]}{\#1^2}\right] \&]}{3 a^2 n}$$

Problem 2638: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{4}}}{a + b x^n} dx$$

Optimal (type 3, 234 leaves, 12 steps):

$$-\frac{4 x^{-n/4}}{a n} - \frac{\sqrt{2} b^{1/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}}\right]}{a^{5/4} n} + \frac{\sqrt{2} b^{1/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}}\right]}{a^{5/4} n} -$$

$$\frac{b^{1/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} - \sqrt{2} a^{1/4} b^{1/4} x^{-n/4}\right]}{\sqrt{2} a^{5/4} n} + \frac{b^{1/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} + \sqrt{2} a^{1/4} b^{1/4} x^{-n/4}\right]}{\sqrt{2} a^{5/4} n}$$

Result (type 7, 59 leaves):

$$\frac{-16 a x^{-n/4} + b \operatorname{RootSum}\left[b + a \#1^4 \&, \frac{n \operatorname{Log}[x] + 4 \operatorname{Log}\left[x^{-n/4} - \#1\right]}{\#1^3}\right] \&]}{4 a^2 n}$$

Problem 2640: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{4n}{3}}}{a + b x^n} dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$-\frac{3 x^{-4 n / 3}}{4 a n} + \frac{3 b x^{-n / 3}}{a^2 n} + \frac{\sqrt{3} b^{4 / 3} \text{ArcTan}\left[\frac{b^{1 / 3}-2 a^{1 / 3} x^{-n / 3}}{\sqrt{3} b^{1 / 3}}\right]}{a^{7 / 3} n} - \frac{b^{4 / 3} \text{Log}\left[b^{1 / 3}+a^{1 / 3} x^{-n / 3}\right]}{a^{7 / 3} n} + \frac{b^{4 / 3} \text{Log}\left[b^{2 / 3}+a^{2 / 3} x^{-2 n / 3}-a^{1 / 3} b^{1 / 3} x^{-n / 3}\right]}{2 a^{7 / 3} n}$$

Result (type 7, 70 leaves):

$$-\frac{1}{12 a^3 n} \left(9 a x^{-4 n / 3} (a - 4 b x^n) + 4 b^2 \text{RootSum}\left[b + a \#1^3 \&, \frac{n \text{Log}[x] + 3 \text{Log}\left[x^{-n / 3} - \#1\right]}{\#1^2} \&\right] \right)$$

Problem 2641: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{5n}{4}}}{a + b x^n} dx$$

Optimal (type 3, 252 leaves, 13 steps):

$$-\frac{4 x^{-5 n / 4}}{5 a n} + \frac{4 b x^{-n / 4}}{a^2 n} + \frac{\sqrt{2} b^{5 / 4} \text{ArcTan}\left[1 - \frac{\sqrt{2} a^{1 / 4} x^{-n / 4}}{b^{1 / 4}}\right]}{a^{9 / 4} n} - \frac{\sqrt{2} b^{5 / 4} \text{ArcTan}\left[1 + \frac{\sqrt{2} a^{1 / 4} x^{-n / 4}}{b^{1 / 4}}\right]}{a^{9 / 4} n} + \frac{b^{5 / 4} \text{Log}\left[\sqrt{b} + \sqrt{a} x^{-n / 2} - \sqrt{2} a^{1 / 4} b^{1 / 4} x^{-n / 4}\right]}{\sqrt{2} a^{9 / 4} n} - \frac{b^{5 / 4} \text{Log}\left[\sqrt{b} + \sqrt{a} x^{-n / 2} + \sqrt{2} a^{1 / 4} b^{1 / 4} x^{-n / 4}\right]}{\sqrt{2} a^{9 / 4} n}$$

Result (type 7, 70 leaves):

$$-\frac{1}{20 a^3 n} \left(16 a x^{-5 n / 4} (a - 5 b x^n) + 5 b^2 \text{RootSum}\left[b + a \#1^4 \&, \frac{n \text{Log}[x] + 4 \text{Log}\left[x^{-n / 4} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 2665: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{(a + b x^n)^3} dx$$

Optimal (type 5, 40 leaves, 1 step):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[3, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a^3 (1+m)}$$

Result (type 5, 100 leaves):

$$\frac{1}{2 a^3 n^2} x^{1+m} \left(\frac{a^2 n}{(a+b x^n)^2} - \frac{a(1+m-2n)}{a+b x^n} + \frac{1}{1+m} \right. \\ \left. (1+m^2+m(2-3n)-3n+2n^2) \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] \right)$$

Problem 2666: Result more than twice size of optimal antiderivative.

$$\int x^m (a+b x^n)^{3/2} dx$$

Optimal (type 5, 55 leaves, 2 steps):

$$\frac{x^{1+m} (a+b x^n)^{5/2} \operatorname{Hypergeometric2F1}\left[1, \frac{5}{2} + \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a(1+m)}$$

Result (type 5, 124 leaves):

$$\left(x^{1+m} \left(2(1+m)(a+b x^n)(2a(1+m+2n)+b(2+2m+n)x^n) + \right. \right. \\ \left. \left. 3a^2 n^2 \sqrt{1+\frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] \right) \right) / \\ \left((1+m)(2+2m+n)(2+2m+3n) \sqrt{a+b x^n} \right)$$

Problem 2670: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{(a+b x^n)^{5/2}} dx$$

Optimal (type 5, 55 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[1, -\frac{3}{2} + \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a(1+m)(a+b x^n)^{3/2}}$$

Result (type 5, 129 leaves):

$$\left(x^{1+m} \left(2(1+m)(a n - (2+2m-3n)(a+b x^n)) + (4+4m^2-8m(-1+n)-8n+3n^2)(a+b x^n) \right. \right. \\ \left. \left. \sqrt{1+\frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] \right) \right) / \left(3a^2(1+m)n^2(a+b x^n)^{3/2} \right)$$

Problem 2693: Result unnecessarily involves higher level functions.

$$\int \frac{x^m}{(a + b x^3 (1+m))^{1/3}} dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x^{1+m}}{(a+b x^3 (1+m))^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} (1+m)} - \frac{\text{Log}\left[b^{1/3} x^{1+m} - (a + b x^3 (1+m))^{1/3}\right]}{2 b^{1/3} (1+m)}$$

Result (type 5, 68 leaves):

$$\frac{x^{1+m} \left(\frac{a+b x^{3+3m}}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{b x^{3+3m}}{a}\right]}{(1+m) (a + b x^{3+3m})^{1/3}}$$

Problem 2694: Result unnecessarily involves higher level functions.

$$\int x^m (a + b x^{-\frac{3}{2} (1+m)})^{2/3} dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$\frac{x^{1+m} (a + b x^{-\frac{3}{2} (1+m)})^{2/3}}{1+m} - \frac{2 b^{2/3} \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x^{\frac{1}{2} (-1-m)}}{(a+b x^{-\frac{3}{2} (1+m)})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (1+m)} + \frac{b^{2/3} \text{Log}\left[b^{1/3} x^{\frac{1}{2} (-1-m)} - (a + b x^{-\frac{3}{2} (1+m)})^{1/3}\right]}{1+m}$$

Result (type 5, 73 leaves):

$$\frac{x^{1+m} (a + b x^{-\frac{3}{2} (1+m)})^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{b x^{-\frac{3}{2} (1+m)}}{a}\right]}{(1+m) \left(1 + \frac{b x^{-\frac{3}{2} (1+m)}}{a}\right)^{2/3}}$$

Problem 2695: Result unnecessarily involves higher level functions.

$$\int \frac{x^{-1+\frac{n}{3}}}{(a + b x^n)^{1/3}} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x^{n/3}}{(a+b x^n)^{1/3}}}{\sqrt{3}}\right]}{b^{1/3} n} - \frac{3 \text{Log}\left[b^{1/3} x^{n/3} - (a + b x^n)^{1/3}\right]}{2 b^{1/3} n}$$

Result (type 5, 57 leaves):

$$\frac{3 x^{n/3} \left(\frac{a+bx^n}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{bx^n}{a}\right]}{n (a+bx^n)^{1/3}}$$

Problem 2696: Result unnecessarily involves higher level functions.

$$\int x^{-1-\frac{2n}{3}} (a+bx^n)^{2/3} dx$$

Optimal (type 3, 114 leaves, 3 steps):

$$-\frac{3 x^{-2n/3} (a+bx^n)^{2/3}}{2n} + \frac{\sqrt{3} b^{2/3} \text{ArcTan}\left[\frac{1+2b^{1/3}x^{n/3}}{(a+bx^n)^{1/3}}\right]}{n} - \frac{3 b^{2/3} \text{Log}\left[b^{1/3}x^{n/3} - (a+bx^n)^{1/3}\right]}{2n}$$

Result (type 5, 71 leaves):

$$-\frac{1}{2n (a+bx^n)^{1/3}} 3 x^{-2n/3} \left(a+bx^n - 2bx^n \left(1 + \frac{bx^n}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{bx^n}{a}\right]\right)$$

Problem 2698: Result unnecessarily involves higher level functions.

$$\int (a+bx^n)^{-4-\frac{1}{n}} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{x (a+bx^n)^{-3-\frac{1}{n}}}{a (1+3n)} + \frac{3nx (a+bx^n)^{-2-\frac{1}{n}}}{a^2 (1+5n+6n^2)} + \frac{6n^2x (a+bx^n)^{-1-\frac{1}{n}}}{a^3 (1+n) (1+2n) (1+3n)} + \frac{6n^3x (a+bx^n)^{-1/n}}{a^4 (1+n) (1+2n) (1+3n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a+bx^n)^{-1/n} \left(1 + \frac{bx^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[4 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{bx^n}{a}\right]}{a^4}$$

Problem 2699: Result unnecessarily involves higher level functions.

$$\int (a+bx^n)^{-3-\frac{1}{n}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$\frac{x (a+bx^n)^{-2-\frac{1}{n}}}{a (1+2n)} + \frac{2nx (a+bx^n)^{-1-\frac{1}{n}}}{a^2 (1+n) (1+2n)} + \frac{2n^2x (a+bx^n)^{-1/n}}{a^3 (1+n) (1+2n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a+bx^n)^{-1/n} \left(1 + \frac{bx^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[3 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{bx^n}{a}\right]}{a^3}$$

Problem 2700: Result unnecessarily involves higher level functions.

$$\int (a + b x^n)^{-2 - \frac{1}{n}} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{x (a + b x^n)^{-1 - \frac{1}{n}}}{a (1 + n)} + \frac{n x (a + b x^n)^{-1/n}}{a^2 (1 + n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a + b x^n)^{-1/n} \left(1 + \frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[2 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^2}$$

Problem 2718: Result more than twice size of optimal antiderivative.

$$\int x^{-1-9n} (a + b x^n)^8 dx$$

Optimal (type 3, 24 leaves, 1 step):

$$-\frac{x^{-9n} (a + b x^n)^9}{9 a n}$$

Result (type 3, 111 leaves):

$$-\frac{1}{9n} x^{-9n} (a^8 + 9 a^7 b x^n + 36 a^6 b^2 x^{2n} + 84 a^5 b^3 x^{3n} + 126 a^4 b^4 x^{4n} + 126 a^3 b^5 x^{5n} + 84 a^2 b^6 x^{6n} + 36 a b^7 x^{7n} + 9 b^8 x^{8n})$$

Problem 2720: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^8}{x^{28}} dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{(a + b x^3)^9}{27 a x^{27}}$$

Result (type 1, 108 leaves):

$$-\frac{a^8}{27 x^{27}} - \frac{a^7 b}{3 x^{24}} - \frac{4 a^6 b^2}{3 x^{21}} - \frac{28 a^5 b^3}{9 x^{18}} - \frac{14 a^4 b^4}{3 x^{15}} - \frac{14 a^3 b^5}{3 x^{12}} - \frac{28 a^2 b^6}{9 x^9} - \frac{4 a b^7}{3 x^6} - \frac{b^8}{3 x^3}$$

Problem 2723: Result unnecessarily involves higher level functions.

$$\int (a + b x^n)^{-\frac{1+4n}{n}} dx$$

Optimal (type 3, 147 leaves, 4 steps):

$$\frac{x (a + b x^n)^{-3-\frac{1}{n}}}{a (1 + 3 n)} + \frac{3 n x (a + b x^n)^{-2-\frac{1}{n}}}{a^2 (1 + 5 n + 6 n^2)} + \frac{6 n^3 x (a + b x^n)^{-1/n}}{a^4 (1 + n) (1 + 2 n) (1 + 3 n)} + \frac{6 n^2 x (a + b x^n)^{-\frac{1+n}{n}}}{a^3 (1 + n) (1 + 2 n) (1 + 3 n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a + b x^n)^{-1/n} \left(1 + \frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[4 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^4}$$

Problem 2724: Result unnecessarily involves higher level functions.

$$\int (a + b x^n)^{-\frac{1+3n}{n}} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$\frac{x (a + b x^n)^{-2-\frac{1}{n}}}{a (1 + 2 n)} + \frac{2 n^2 x (a + b x^n)^{-1/n}}{a^3 (1 + n) (1 + 2 n)} + \frac{2 n x (a + b x^n)^{-\frac{1+n}{n}}}{a^2 (1 + n) (1 + 2 n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a + b x^n)^{-1/n} \left(1 + \frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[3 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^3}$$

Problem 2725: Result unnecessarily involves higher level functions.

$$\int (a + b x^n)^{-\frac{1+2n}{n}} dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{n x (a + b x^n)^{-1/n}}{a^2 (1 + n)} + \frac{x (a + b x^n)^{-\frac{1+n}{n}}}{a (1 + n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a + b x^n)^{-1/n} \left(1 + \frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[2 + \frac{1}{n}, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b x^n}{a}\right]}{a^2}$$

Problem 2756: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{2n}{3}}}{a + b x^n} dx$$

Optimal (type 3, 222 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{3 (c x)^{-2 n / 3}}{2 a c n} + \frac{\sqrt{3} b^{2 / 3} x^{2 n / 3} (c x)^{-2 n / 3} \operatorname{ArcTan}\left[\frac{a^{1 / 3}-2 b^{1 / 3} x^{n / 3}}{\sqrt{3} a^{1 / 3}}\right]}{a^{5 / 3} c n} - \\
 & \frac{b^{2 / 3} x^{2 n / 3} (c x)^{-2 n / 3} \operatorname{Log}\left[a^{1 / 3}+b^{1 / 3} x^{n / 3}\right]}{a^{5 / 3} c n} + \frac{b^{2 / 3} x^{2 n / 3} (c x)^{-2 n / 3} \operatorname{Log}\left[a^{2 / 3}-a^{1 / 3} b^{1 / 3} x^{n / 3}+b^{2 / 3} x^{2 n / 3}\right]}{2 a^{5 / 3} c n}
 \end{aligned}$$

Result (type 7, 72 leaves):

$$\frac{(c x)^{-2 n / 3} \left(-9 a + 2 b x^{2 n / 3} \operatorname{RootSum}\left[b + a \#1^3 \&, \frac{n \operatorname{Log}[x] + 3 \operatorname{Log}\left[x^{-n / 3} - \#1 \right]}{\#1} \& \right] \right)}{6 a^2 c n}$$

Problem 2757: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{3 n}{4}}}{a+b x^n} dx$$

Optimal (type 3, 317 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{4 (c x)^{-3 n / 4}}{3 a c n} + \frac{\sqrt{2} b^{3 / 4} x^{3 n / 4} (c x)^{-3 n / 4} \operatorname{ArcTan}\left[1-\frac{\sqrt{2} b^{1 / 4} x^{n / 4}}{a^{1 / 4}}\right]}{a^{7 / 4} c n} - \\
 & \frac{\sqrt{2} b^{3 / 4} x^{3 n / 4} (c x)^{-3 n / 4} \operatorname{ArcTan}\left[1+\frac{\sqrt{2} b^{1 / 4} x^{n / 4}}{a^{1 / 4}}\right]}{a^{7 / 4} c n} + \\
 & \frac{b^{3 / 4} x^{3 n / 4} (c x)^{-3 n / 4} \operatorname{Log}\left[\sqrt{a}-\sqrt{2} a^{1 / 4} b^{1 / 4} x^{n / 4}+\sqrt{b} x^{n / 2}\right]}{\sqrt{2} a^{7 / 4} c n} - \\
 & \frac{b^{3 / 4} x^{3 n / 4} (c x)^{-3 n / 4} \operatorname{Log}\left[\sqrt{a}+\sqrt{2} a^{1 / 4} b^{1 / 4} x^{n / 4}+\sqrt{b} x^{n / 2}\right]}{\sqrt{2} a^{7 / 4} c n}
 \end{aligned}$$

Result (type 7, 72 leaves):

$$\frac{1}{12 a^2 c n} (c x)^{-3 n / 4} \left(-16 a + 3 b x^{3 n / 4} \operatorname{RootSum}\left[b + a \#1^4 \&, \frac{n \operatorname{Log}[x] + 4 \operatorname{Log}\left[x^{-n / 4} - \#1 \right]}{\#1} \& \right] \right)$$

Problem 2760: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{n}{3}}}{a+b x^n} dx$$

Optimal (type 3, 220 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{3 (c x)^{-n / 3}}{a c n} - \frac{\sqrt{3} b^{1 / 3} x^{n / 3} (c x)^{-n / 3} \operatorname{ArcTan}\left[\frac{b^{1 / 3}-2 a^{1 / 3} x^{n / 3}}{\sqrt{3} b^{1 / 3}}\right]}{a^{4 / 3} c n} + \\
 & \frac{b^{1 / 3} x^{n / 3} (c x)^{-n / 3} \operatorname{Log}\left[b^{1 / 3}+a^{1 / 3} x^{n / 3}\right]}{a^{4 / 3} c n} - \frac{b^{1 / 3} x^{n / 3} (c x)^{-n / 3} \operatorname{Log}\left[b^{2 / 3}+a^{2 / 3} x^{-2 n / 3}-a^{1 / 3} b^{1 / 3} x^{-n / 3}\right]}{2 a^{4 / 3} c n}
 \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{(c x)^{-n/3} \left(-9 a + b x^{n/3} \operatorname{RootSum}\left[b + a \#1^3 \&, \frac{n \operatorname{Log}[x] + 3 \operatorname{Log}\left[x^{-n/3} - \#1 \right]}{\#1^2} \& \right] \right)}{3 a^2 c n}$$

Problem 2761: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{n}{4}}}{a + b x^n} dx$$

Optimal (type 3, 315 leaves, 13 steps):

$$\begin{aligned} & -\frac{4 (c x)^{-n/4}}{a c n} - \frac{\sqrt{2} b^{1/4} x^{n/4} (c x)^{-n/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}} \right]}{a^{5/4} c n} + \\ & \frac{\sqrt{2} b^{1/4} x^{n/4} (c x)^{-n/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}} \right]}{a^{5/4} c n} - \\ & \frac{b^{1/4} x^{n/4} (c x)^{-n/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} - \sqrt{2} a^{1/4} b^{1/4} x^{-n/4} \right]}{\sqrt{2} a^{5/4} c n} + \\ & \frac{b^{1/4} x^{n/4} (c x)^{-n/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} + \sqrt{2} a^{1/4} b^{1/4} x^{-n/4} \right]}{\sqrt{2} a^{5/4} c n} \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{(c x)^{-n/4} \left(-16 a + b x^{n/4} \operatorname{RootSum}\left[b + a \#1^4 \&, \frac{n \operatorname{Log}[x] + 4 \operatorname{Log}\left[x^{-n/4} - \#1 \right]}{\#1^3} \& \right] \right)}{4 a^2 c n}$$

Problem 2763: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{4n}{3}}}{a + b x^n} dx$$

Optimal (type 3, 246 leaves, 11 steps):

$$\begin{aligned} & -\frac{3 (c x)^{-4n/3}}{4 a c n} + \frac{3 b x^n (c x)^{-4n/3}}{a^2 c n} + \frac{\sqrt{3} b^{4/3} x^{4n/3} (c x)^{-4n/3} \operatorname{ArcTan}\left[\frac{b^{1/3} - 2 a^{1/3} x^{-n/3}}{\sqrt{3} b^{1/3}} \right]}{a^{7/3} c n} - \\ & \frac{b^{4/3} x^{4n/3} (c x)^{-4n/3} \operatorname{Log}\left[b^{1/3} + a^{1/3} x^{-n/3} \right]}{a^{7/3} c n} + \frac{b^{4/3} x^{4n/3} (c x)^{-4n/3} \operatorname{Log}\left[b^{2/3} + a^{2/3} x^{-2n/3} - a^{1/3} b^{1/3} x^{-n/3} \right]}{2 a^{7/3} c n} \end{aligned}$$

Result (type 7, 82 leaves):

$$\frac{1}{12 a^3 c n} (c x)^{-4n/3} \left(-9 a (a - 4 b x^n) - 4 b^2 x^{4n/3} \operatorname{RootSum}\left[b + a \#1^3 \&, \frac{n \operatorname{Log}[x] + 3 \operatorname{Log}\left[x^{-n/3} - \#1 \right]}{\#1^2} \& \right] \right)$$

Problem 2764: Result is not expressed in closed-form.

$$\int \frac{(c x)^{-1-\frac{5n}{4}}}{a+b x^n} dx$$

Optimal (type 3, 341 leaves, 14 steps):

$$\begin{aligned} & -\frac{4 (c x)^{-5n/4}}{5 a c n} + \frac{4 b x^n (c x)^{-5n/4}}{a^2 c n} + \frac{\sqrt{2} b^{5/4} x^{5n/4} (c x)^{-5n/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}}\right]}{a^{9/4} c n} \\ & - \frac{\sqrt{2} b^{5/4} x^{5n/4} (c x)^{-5n/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} a^{1/4} x^{-n/4}}{b^{1/4}}\right]}{a^{9/4} c n} + \\ & - \frac{b^{5/4} x^{5n/4} (c x)^{-5n/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} - \sqrt{2} a^{1/4} b^{1/4} x^{-n/4}\right]}{\sqrt{2} a^{9/4} c n} \\ & - \frac{b^{5/4} x^{5n/4} (c x)^{-5n/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{a} x^{-n/2} + \sqrt{2} a^{1/4} b^{1/4} x^{-n/4}\right]}{\sqrt{2} a^{9/4} c n} \end{aligned}$$

Result (type 7, 82 leaves):

$$\begin{aligned} & \frac{1}{20 a^3 c n} \\ & (c x)^{-5n/4} \left(-16 a (a - 5 b x^n) - 5 b^2 x^{5n/4} \operatorname{RootSum}\left[b + a \mp 1^4 \&, \frac{n \operatorname{Log}[x] + 4 \operatorname{Log}\left[x^{-n/4} - \mp 1\right]}{\mp 1^3} \& \right] \right) \end{aligned}$$

Problem 2792: Result unnecessarily involves higher level functions.

$$\int (c x)^{-1-2n-np} (a+b x^n)^p dx$$

Optimal (type 3, 79 leaves, 2 steps):

$$-\frac{(c x)^{-n(2+p)} (a+b x^n)^{1+p}}{a c n (1+p)} + \frac{(c x)^{-n(2+p)} (a+b x^n)^{2+p}}{a^2 c n (1+p) (2+p)}$$

Result (type 5, 69 leaves):

$$-\frac{1}{n(2+p)} x (c x)^{-1-n(2+p)} (a+b x^n)^p \left(1 + \frac{b x^n}{a}\right)^{-p} \operatorname{Hypergeometric2F1}\left[-2-p, -p, -1-p, -\frac{b x^n}{a}\right]$$

Problem 2793: Result unnecessarily involves higher level functions.

$$\int (c x)^{-1-3n-np} (a+b x^n)^p dx$$

Optimal (type 3, 127 leaves, 3 steps):

$$-\frac{(c x)^{-n(3+p)} (a+b x^n)^{1+p}}{a c n (1+p)} + \frac{2 (c x)^{-n(3+p)} (a+b x^n)^{2+p}}{a^2 c n (1+p) (2+p)} - \frac{2 (c x)^{-n(3+p)} (a+b x^n)^{3+p}}{a^3 c n (1+p) (2+p) (3+p)}$$

Result (type 5, 69 leaves):

$$-\frac{1}{n(3+p)} x (c x)^{-1-n(3+p)} (a + b x^n)^p \left(1 + \frac{b x^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left[-3-p, -p, -2-p, -\frac{b x^n}{a}\right]$$

Problem 2794: Result unnecessarily involves higher level functions.

$$\int (c x)^{-1-4n-np} (a + b x^n)^p dx$$

Optimal (type 3, 179 leaves, 4 steps):

$$-\frac{(c x)^{-n(4+p)} (a + b x^n)^{1+p}}{a c n (1+p)} + \frac{3 (c x)^{-n(4+p)} (a + b x^n)^{2+p}}{a^2 c n (1+p) (2+p)} - \frac{6 (c x)^{-n(4+p)} (a + b x^n)^{3+p}}{a^3 c n (1+p) (2+p) (3+p)} + \frac{6 (c x)^{-n(4+p)} (a + b x^n)^{4+p}}{a^4 c n (1+p) (2+p) (3+p) (4+p)}$$

Result (type 5, 69 leaves):

$$-\frac{1}{n(4+p)} x (c x)^{-1-n(4+p)} (a + b x^n)^p \left(1 + \frac{b x^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left[-4-p, -p, -3-p, -\frac{b x^n}{a}\right]$$

Problem 2834: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^2) dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$\frac{a (c + d x)^4}{4 d} + \frac{b (c + d x)^6}{6 d}$$

Result (type 1, 77 leaves):

$$\frac{1}{12} x (2 c + d x) (3 a (2 c^2 + 2 c d x + d^2 x^2) + 2 b (3 c^4 + 6 c^3 d x + 7 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4))$$

Problem 2835: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^2)^2 dx$$

Optimal (type 1, 51 leaves, 4 steps):

$$\frac{a^2 (c + d x)^4}{4 d} + \frac{a b (c + d x)^6}{3 d} + \frac{b^2 (c + d x)^8}{8 d}$$

Result (type 1, 172 leaves):

$$\begin{aligned}
 & c^3 (a + b c^2)^2 x + \frac{1}{2} c^2 (3 a^2 + 10 a b c^2 + 7 b^2 c^4) d x^2 + \\
 & \frac{1}{3} c (3 a^2 + 20 a b c^2 + 21 b^2 c^4) d^2 x^3 + \frac{1}{4} (a^2 + 20 a b c^2 + 35 b^2 c^4) d^3 x^4 + \\
 & b c (2 a + 7 b c^2) d^4 x^5 + \frac{1}{6} b (2 a + 21 b c^2) d^5 x^6 + b^2 c d^6 x^7 + \frac{1}{8} b^2 d^7 x^8
 \end{aligned}$$

Problem 2836: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^2)^3 dx$$

Optimal (type 1, 48 leaves, 4 steps):

$$-\frac{a (a + b (c + d x)^2)^4}{8 b^2 d} + \frac{(a + b (c + d x)^2)^5}{10 b^2 d}$$

Result (type 1, 249 leaves):

$$\begin{aligned}
 & c^3 (a + b c^2)^3 x + \frac{3}{2} c^2 (a + b c^2)^2 (a + 3 b c^2) d x^2 + \\
 & c (a^3 + 10 a^2 b c^2 + 21 a b^2 c^4 + 12 b^3 c^6) d^2 x^3 + \frac{1}{4} (a^3 + 30 a^2 b c^2 + 105 a b^2 c^4 + 84 b^3 c^6) d^3 x^4 + \\
 & \frac{3}{5} b c (5 a^2 + 35 a b c^2 + 42 b^2 c^4) d^4 x^5 + \frac{1}{2} b (a^2 + 21 a b c^2 + 42 b^2 c^4) d^5 x^6 + \\
 & 3 b^2 c (a + 4 b c^2) d^6 x^7 + \frac{3}{8} b^2 (a + 12 b c^2) d^7 x^8 + b^3 c d^8 x^9 + \frac{1}{10} b^3 d^9 x^{10}
 \end{aligned}$$

Problem 2846: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^3) dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$\frac{a (c + d x)^4}{4 d} + \frac{b (c + d x)^7}{7 d}$$

Result (type 1, 98 leaves):

$$\begin{aligned}
 & c^3 (a + b c^3) x + \frac{3}{2} c^2 (a + 2 b c^3) d x^2 + c (a + 5 b c^3) d^2 x^3 + \\
 & \frac{1}{4} (a + 20 b c^3) d^3 x^4 + 3 b c^2 d^4 x^5 + b c d^5 x^6 + \frac{1}{7} b d^6 x^7
 \end{aligned}$$

Problem 2847: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^3)^2 dx$$

Optimal (type 1, 51 leaves, 3 steps):

$$\frac{a^2 (c + d x)^4}{4 d} + \frac{2 a b (c + d x)^7}{7 d} + \frac{b^2 (c + d x)^{10}}{10 d}$$

Result (type 1, 203 leaves):

$$\begin{aligned} & c^3 (a + b c^3)^2 x + \frac{3}{2} c^2 (a^2 + 4 a b c^3 + 3 b^2 c^6) d x^2 + c (a^2 + 10 a b c^3 + 12 b^2 c^6) d^2 x^3 + \\ & \frac{1}{4} (a^2 + 40 a b c^3 + 84 b^2 c^6) d^3 x^4 + \frac{6}{5} b c^2 (5 a + 21 b c^3) d^4 x^5 + \\ & b c (2 a + 21 b c^3) d^5 x^6 + \frac{2}{7} b (a + 42 b c^3) d^6 x^7 + \frac{9}{2} b^2 c^2 d^7 x^8 + b^2 c d^8 x^9 + \frac{1}{10} b^2 d^9 x^{10} \end{aligned}$$

Problem 2848: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^3)^3 dx$$

Optimal (type 1, 71 leaves, 3 steps):

$$\frac{a^3 (c + d x)^4}{4 d} + \frac{3 a^2 b (c + d x)^7}{7 d} + \frac{3 a b^2 (c + d x)^{10}}{10 d} + \frac{b^3 (c + d x)^{13}}{13 d}$$

Result (type 1, 323 leaves):

$$\begin{aligned} & c^3 (a + b c^3)^3 x + \frac{3}{2} c^2 (a + b c^3)^2 (a + 4 b c^3) d x^2 + c (a^3 + 15 a^2 b c^3 + 36 a b^2 c^6 + 22 b^3 c^9) d^2 x^3 + \\ & \frac{1}{4} (a^3 + 60 a^2 b c^3 + 252 a b^2 c^6 + 220 b^3 c^9) d^3 x^4 + \frac{9}{5} b c^2 (5 a^2 + 42 a b c^3 + 55 b^2 c^6) d^4 x^5 + \\ & 3 b c (a^2 + 21 a b c^3 + 44 b^2 c^6) d^5 x^6 + \frac{3}{7} b (a^2 + 84 a b c^3 + 308 b^2 c^6) d^6 x^7 + \frac{9}{2} b^2 c^2 (3 a + 22 b c^3) d^7 x^8 + \\ & b^2 c (3 a + 55 b c^3) d^8 x^9 + \frac{1}{10} b^2 (3 a + 220 b c^3) d^9 x^{10} + 6 b^3 c^2 d^{10} x^{11} + b^3 c d^{11} x^{12} + \frac{1}{13} b^3 d^{12} x^{13} \end{aligned}$$

Problem 2849: Result more than twice size of optimal antiderivative.

$$\int (c e + d e x)^3 (a + b (c + d x)^3) dx$$

Optimal (type 1, 37 leaves, 3 steps):

$$\frac{a e^3 (c + d x)^4}{4 d} + \frac{b e^3 (c + d x)^7}{7 d}$$

Result (type 1, 102 leaves):

$$\begin{aligned} & e^3 \left(c^3 (a + b c^3) x + \frac{3}{2} c^2 (a + 2 b c^3) d x^2 + \right. \\ & \left. c (a + 5 b c^3) d^2 x^3 + \frac{1}{4} (a + 20 b c^3) d^3 x^4 + 3 b c^2 d^4 x^5 + b c d^5 x^6 + \frac{1}{7} b d^6 x^7 \right) \end{aligned}$$

Problem 2850: Result more than twice size of optimal antiderivative.

$$\int (c e + d e x)^3 (a + b (c + d x)^3)^2 dx$$

Optimal (type 1, 60 leaves, 3 steps):

$$\frac{a^2 e^3 (c + d x)^4}{4 d} + \frac{2 a b e^3 (c + d x)^7}{7 d} + \frac{b^2 e^3 (c + d x)^{10}}{10 d}$$

Result (type 1, 207 leaves):

$$e^3 \left(c^3 (a + b c^3)^2 x + \frac{3}{2} c^2 (a^2 + 4 a b c^3 + 3 b^2 c^6) d x^2 + c (a^2 + 10 a b c^3 + 12 b^2 c^6) d^2 x^3 + \frac{1}{4} (a^2 + 40 a b c^3 + 84 b^2 c^6) d^3 x^4 + \frac{6}{5} b c^2 (5 a + 21 b c^3) d^4 x^5 + b c (2 a + 21 b c^3) d^5 x^6 + \frac{2}{7} b (a + 42 b c^3) d^6 x^7 + \frac{9}{2} b^2 c^2 d^7 x^8 + b^2 c d^8 x^9 + \frac{1}{10} b^2 d^9 x^{10} \right)$$

Problem 2851: Result more than twice size of optimal antiderivative.

$$\int (c e + d e x)^3 (a + b (c + d x)^3)^3 dx$$

Optimal (type 1, 83 leaves, 3 steps):

$$\frac{a^3 e^3 (c + d x)^4}{4 d} + \frac{3 a^2 b e^3 (c + d x)^7}{7 d} + \frac{3 a b^2 e^3 (c + d x)^{10}}{10 d} + \frac{b^3 e^3 (c + d x)^{13}}{13 d}$$

Result (type 1, 327 leaves):

$$e^3 \left(c^3 (a + b c^3)^3 x + \frac{3}{2} c^2 (a + b c^3)^2 (a + 4 b c^3) d x^2 + c (a^3 + 15 a^2 b c^3 + 36 a b^2 c^6 + 22 b^3 c^9) d^2 x^3 + \frac{1}{4} (a^3 + 60 a^2 b c^3 + 252 a b^2 c^6 + 220 b^3 c^9) d^3 x^4 + \frac{9}{5} b c^2 (5 a^2 + 42 a b c^3 + 55 b^2 c^6) d^4 x^5 + 3 b c (a^2 + 21 a b c^3 + 44 b^2 c^6) d^5 x^6 + \frac{3}{7} b (a^2 + 84 a b c^3 + 308 b^2 c^6) d^6 x^7 + \frac{9}{2} b^2 c^2 (3 a + 22 b c^3) d^7 x^8 + b^2 c (3 a + 55 b c^3) d^8 x^9 + \frac{1}{10} b^2 (3 a + 220 b c^3) d^9 x^{10} + 6 b^3 c^2 d^{10} x^{11} + b^3 c d^{11} x^{12} + \frac{1}{13} b^3 d^{12} x^{13} \right)$$

Problem 2904: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b (c + d x)^4) dx$$

Optimal (type 1, 23 leaves, 3 steps):

$$\frac{(a + b (c + d x)^4)^2}{8 b d}$$

Result (type 1, 80 leaves):

$$\frac{1}{8} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left(2 a + b \left(2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) \right)$$

Problem 2905: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \left(a + b (c + d x)^4 \right)^2 dx$$

Optimal (type 1, 23 leaves, 2 steps):

$$\frac{(a + b (c + d x)^4)^3}{12 b d}$$

Result (type 1, 172 leaves):

$$\frac{1}{12} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left(3 a^2 + 3 a b \left(2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) + b^2 \left(3 c^8 + 12 c^7 d x + 34 c^6 d^2 x^2 + 60 c^5 d^3 x^3 + 71 c^4 d^4 x^4 + 56 c^3 d^5 x^5 + 28 c^2 d^6 x^6 + 8 c d^7 x^7 + d^8 x^8 \right) \right)$$

Problem 2906: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \left(a + b (c + d x)^4 \right)^3 dx$$

Optimal (type 1, 23 leaves, 2 steps):

$$\frac{(a + b (c + d x)^4)^4}{16 b d}$$

Result (type 1, 308 leaves):

$$\frac{1}{16} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left(4 a^3 + 6 a^2 b \left(2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) + 4 a b^2 \left(3 c^8 + 12 c^7 d x + 34 c^6 d^2 x^2 + 60 c^5 d^3 x^3 + 71 c^4 d^4 x^4 + 56 c^3 d^5 x^5 + 28 c^2 d^6 x^6 + 8 c d^7 x^7 + d^8 x^8 \right) + b^3 \left(4 c^{12} + 24 c^{11} d x + 100 c^{10} d^2 x^2 + 280 c^9 d^3 x^3 + 566 c^8 d^4 x^4 + 848 c^7 d^5 x^5 + 952 c^6 d^6 x^6 + 800 c^5 d^7 x^7 + 496 c^4 d^8 x^8 + 220 c^3 d^9 x^9 + 66 c^2 d^{10} x^{10} + 12 c d^{11} x^{11} + d^{12} x^{12} \right) \right)$$

Problem 2910: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b (c + d x)^4}} dx$$

Optimal (type 4, 111 leaves, 2 steps):

$$\left(\frac{(\sqrt{a} + \sqrt{b} (c + d x)^2) \sqrt{\frac{a + b (c + d x)^4}{(\sqrt{a} + \sqrt{b} (c + d x)^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} (c + d x)}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} d \sqrt{a + b (c + d x)^4}} \right) /$$

Result (type 4, 90 leaves):

$$\frac{i \sqrt{\frac{a+b(c+dx)^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} d \sqrt{a+b(c+dx)^4}}$$

Problem 2911: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}} dx$$

Optimal (type 4, 154 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right]}{2\sqrt{b}d^2} - \frac{\left(c(\sqrt{a}+\sqrt{b}(c+dx)^2) \sqrt{\frac{a+b(c+dx)^4}{(\sqrt{a}+\sqrt{b}(c+dx)^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}(c+dx)}{a^{1/4}}\right], \frac{1}{2}\right]\right)}{\left(2a^{1/4}b^{1/4}d^2\sqrt{a+b(c+dx)^4}\right)}$$

Result (type 4, 330 leaves):

$$\left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c+dx) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c+dx)}} (i \sqrt{a} + \sqrt{b} (c+dx)^2) \right. \\ \left. \left(\left((-1)^{1/4} a^{1/4} - b^{1/4} c \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c+dx) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c+dx)}}\right], -1\right] - \right. \right. \\ \left. \left. 2 (-1)^{1/4} a^{1/4} \operatorname{EllipticPi}\left[-i, \operatorname{ArcSin}\left[\sqrt{-\frac{i \left((-1)^{1/4} a^{1/4} + b^{1/4} (c+dx) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c+dx)}}\right], -1\right] \right) \right) / \\ \left(a^{1/4} \sqrt{b} d^2 \sqrt{\frac{i \sqrt{a} + \sqrt{b} (c+dx)^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} (c+dx) \right)^2}} \sqrt{a+b(c+dx)^4} \right)$$

Problem 2922: Unable to integrate problem.

$$\int \frac{1}{1+(x^2)^{3/2}} dx$$

Optimal (type 3, 83 leaves, 7 steps):

$$-\frac{x \operatorname{ArcTan}\left[\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right]}{\sqrt{3}\sqrt{x^2}} - \frac{x \operatorname{Log}\left[1+x^2-\sqrt{x^2}\right]}{6\sqrt{x^2}} + \frac{x \operatorname{Log}\left[1+\sqrt{x^2}\right]}{3\sqrt{x^2}}$$

Result (type 8, 13 leaves):

$$\int \frac{1}{1+(x^2)^{3/2}} dx$$

Problem 2926: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x} dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$2\sqrt{a+b\sqrt{cx^2}} - 2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x} dx$$

Problem 2927: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$-\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} - \frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}} + \frac{b^2c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right]}{4a^{3/2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3} dx$$

Problem 2928: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b}\sqrt{c x^2}}{4 x^4} + \frac{5 b^2 c \sqrt{a+b}\sqrt{c x^2}}{96 a^2 x^2} - \\
 & \frac{b c^2 \sqrt{a+b}\sqrt{c x^2}}{24 a (c x^2)^{3/2}} - \frac{5 b^3 c^2 \sqrt{a+b}\sqrt{c x^2}}{64 a^3 \sqrt{c x^2}} + \frac{5 b^4 c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b}\sqrt{c x^2}}{\sqrt{a}}\right]}{64 a^{7/2}}
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b}\sqrt{c x^2}}{x^5} dx$$

Problem 2929: Unable to integrate problem.

$$\int x^4 \sqrt{a+b}\sqrt{c x^2} dx$$

Optimal (type 2, 191 leaves, 3 steps):

$$\begin{aligned}
 & \frac{2 a^4 x^5 (a+b)\sqrt{c x^2}^{3/2}}{3 b^5 (c x^2)^{5/2}} - \frac{8 a^3 x^5 (a+b)\sqrt{c x^2}^{5/2}}{5 b^5 (c x^2)^{5/2}} + \\
 & \frac{12 a^2 x^5 (a+b)\sqrt{c x^2}^{7/2}}{7 b^5 (c x^2)^{5/2}} - \frac{8 a x^5 (a+b)\sqrt{c x^2}^{9/2}}{9 b^5 (c x^2)^{5/2}} + \frac{2 x^5 (a+b)\sqrt{c x^2}^{11/2}}{11 b^5 (c x^2)^{5/2}}
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int x^4 \sqrt{a+b}\sqrt{c x^2} dx$$

Problem 2930: Unable to integrate problem.

$$\int x^2 \sqrt{a+b}\sqrt{c x^2} dx$$

Optimal (type 2, 113 leaves, 3 steps):

$$\frac{2 a^2 x^3 (a+b)\sqrt{c x^2}^{3/2}}{3 b^3 (c x^2)^{3/2}} - \frac{4 a x^3 (a+b)\sqrt{c x^2}^{5/2}}{5 b^3 (c x^2)^{3/2}} + \frac{2 x^3 (a+b)\sqrt{c x^2}^{7/2}}{7 b^3 (c x^2)^{3/2}}$$

Result (type 8, 23 leaves):

$$\int x^2 \sqrt{a+b}\sqrt{c x^2} dx$$

Problem 2932: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^2}}}{x^2} dx$$

Optimal (type 3, 67 leaves, 4 steps):

$$-\frac{\sqrt{a + b \sqrt{c x^2}}}{x} - \frac{b \sqrt{c x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sqrt{c x^2}}}{\sqrt{a}}\right]}{\sqrt{a} x}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b \sqrt{c x^2}}}{x^2} dx$$

Problem 2933: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^2}}}{x^4} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$-\frac{\sqrt{a + b \sqrt{c x^2}}}{3 x^3} + \frac{b^2 c \sqrt{a + b \sqrt{c x^2}}}{8 a^2 x} - \frac{b (c x^2)^{3/2} \sqrt{a + b \sqrt{c x^2}}}{12 a c x^5} - \frac{b^3 (c x^2)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sqrt{c x^2}}}{\sqrt{a}}\right]}{8 a^{5/2} x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b \sqrt{c x^2}}}{x^4} dx$$

Problem 2934: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^2}}}{x^6} dx$$

Optimal (type 3, 219 leaves, 8 steps):

$$-\frac{\sqrt{a + b \sqrt{c x^2}}}{5 x^5} + \frac{7 b^2 c \sqrt{a + b \sqrt{c x^2}}}{240 a^2 x^3} + \frac{7 b^4 c^2 \sqrt{a + b \sqrt{c x^2}}}{128 a^4 x} - \frac{b (c x^2)^{5/2} \sqrt{a + b \sqrt{c x^2}}}{40 a c^2 x^9} - \frac{7 b^3 (c x^2)^{5/2} \sqrt{a + b \sqrt{c x^2}}}{192 a^3 c x^7} - \frac{7 b^5 (c x^2)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sqrt{c x^2}}}{\sqrt{a}}\right]}{128 a^{9/2} x^5}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6} dx$$

Problem 2935: Unable to integrate problem.

$$\int x^8 \sqrt{a+b(cx^2)^{3/2}} dx$$

Optimal (type 2, 113 leaves, 4 steps):

$$\frac{2a^2x^9(a+b(cx^2)^{3/2})^{3/2}}{9b^3(cx^2)^{9/2}} - \frac{4ax^9(a+b(cx^2)^{3/2})^{5/2}}{15b^3(cx^2)^{9/2}} + \frac{2x^9(a+b(cx^2)^{3/2})^{7/2}}{21b^3(cx^2)^{9/2}}$$

Result (type 8, 23 leaves):

$$\int x^8 \sqrt{a+b(cx^2)^{3/2}} dx$$

Problem 2936: Unable to integrate problem.

$$\int x^5 \sqrt{a+b(cx^2)^{3/2}} dx$$

Optimal (type 2, 56 leaves, 4 steps):

$$-\frac{2a(a+b(cx^2)^{3/2})^{3/2}}{9b^2c^3} + \frac{2(a+b(cx^2)^{3/2})^{5/2}}{15b^2c^3}$$

Result (type 8, 23 leaves):

$$\int x^5 \sqrt{a+b(cx^2)^{3/2}} dx$$

Problem 2938: Unable to integrate problem.

$$\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{2}{3} \sqrt{a+b(cx^2)^{3/2}} - \frac{2}{3} \sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{a}} \right]$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b (c x^2)^{3/2}}}{x} dx$$

Problem 2939: Unable to integrate problem.

$$\int \frac{\sqrt{a + b (c x^2)^{3/2}}}{x^4} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$-\frac{\sqrt{a + b (c x^2)^{3/2}}}{3 x^3} - \frac{b (c x^2)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b (c x^2)^{3/2}}}{\sqrt{a}}\right]}{3 \sqrt{a} x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b (c x^2)^{3/2}}}{x^4} dx$$

Problem 2940: Result unnecessarily involves higher level functions.

$$\int x^3 \sqrt{a + b (c x^2)^{3/2}} dx$$

Optimal (type 4, 340 leaves, 4 steps):

$$\frac{2}{11} x^4 \sqrt{a + b (c x^2)^{3/2}} + \frac{6 a \sqrt{c x^2} \sqrt{a + b (c x^2)^{3/2}}}{55 b c^2} - \left(4 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}\right], -7 - 4 \sqrt{3}\right] \right) / \left(55 b^{4/3} c^2 \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right)$$

Result (type 5, 132 leaves):

$$\left(16 a b c^2 x^4 + 6 a^2 \sqrt{c x^2} + 10 b^2 c^3 x^6 \sqrt{c x^2} - 6 a^2 \sqrt{c x^2} \sqrt{\frac{a + b (c x^2)^{3/2}}{a}} \right. \\ \left. \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{b (c x^2)^{3/2}}{a}\right] \right) / \left(55 b c^2 \sqrt{a + b (c x^2)^{3/2}} \right)$$

Problem 2941: Unable to integrate problem.

$$\int \sqrt{a + b (c x^2)^{3/2}} dx$$

Optimal (type 4, 306 leaves, 3 steps):

$$\frac{2}{5} x \sqrt{a + b (c x^2)^{3/2}} + \left(2 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a x \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}\right], -7 - 4 \sqrt{3}\right] \right) / \\ \left(5 b^{1/3} \sqrt{c x^2} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right)$$

Result (type 8, 19 leaves):

$$\int \sqrt{a + b (c x^2)^{3/2}} dx$$

Problem 2942: Unable to integrate problem.

$$\int \frac{\sqrt{a + b (c x^2)^{3/2}}}{x^3} dx$$

Optimal (type 4, 298 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b(c x^2)^{3/2}}}{2 x^2} + \left(3^{3/4} \sqrt{2+\sqrt{3}} b^{2/3} c \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \quad \left(2 \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^3} dx$$

Problem 2943: Unable to integrate problem.

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^6} dx$$

Optimal (type 4, 352 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b(c x^2)^{3/2}}}{5 x^5} - \frac{3 b (c x^2)^{5/2} \sqrt{a+b(c x^2)^{3/2}}}{20 a c x^7} - \\
 & \quad \left(3^{3/4} \sqrt{2+\sqrt{3}} b^{5/3} (c x^2)^{5/2} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \quad \left(20 a x^5 \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^6} dx$$

Problem 2944: Unable to integrate problem.

$$\int x^4 \sqrt{a + b (c x^2)^{3/2}} dx$$

Optimal (type 4, 709 leaves, 6 steps):

$$\begin{aligned} & \frac{2}{13} x^5 \sqrt{a + b (c x^2)^{3/2}} + \frac{6 a c x^7 \sqrt{a + b (c x^2)^{3/2}}}{91 b (c x^2)^{5/2}} - \frac{24 a^2 x^5 \sqrt{a + b (c x^2)^{3/2}}}{91 b^{5/3} (c x^2)^{5/2} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)} + \\ & \left(12 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} x^5 \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\ & \quad \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(91 b^{5/3} (c x^2)^{5/2} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} \sqrt{c x^2})}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right) - \\ & \left(8 \sqrt{2} 3^{3/4} a^{7/3} x^5 \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\ & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(91 b^{5/3} (c x^2)^{5/2} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} \sqrt{c x^2})}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right) \end{aligned}$$

Result (type 8, 23 leaves):

$$\int x^4 \sqrt{a + b (c x^2)^{3/2}} dx$$

Problem 2945: Result unnecessarily involves higher level functions.

$$\int x \sqrt{a + b (c x^2)^{3/2}} dx$$

Optimal (type 4, 642 leaves, 5 steps):

$$\frac{2}{7} x^2 \sqrt{a + b (c x^2)^{3/2}} + \frac{6 a \sqrt{a + b (c x^2)^{3/2}}}{7 b^{2/3} c \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)} -$$

$$\left(3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 b^{2/3} c \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right) +$$

$$\left(2 \sqrt{2} 3^{3/4} a^{4/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 b^{2/3} c \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a + b (c x^2)^{3/2}} \right)$$

Result (type 5, 89 leaves):

$$\left(x^2 \left(4 (a + b (c x^2)^{3/2}) + 3 a \sqrt{\frac{a + b (c x^2)^{3/2}}{a}} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{b (c x^2)^{3/2}}{a} \right] \right) \right) /$$

$$\left(14 \sqrt{a + b (c x^2)^{3/2}} \right)$$

Problem 2946: Unable to integrate problem.

$$\int \frac{\sqrt{a + b (c x^2)^{3/2}}}{x^2} dx$$

Optimal (type 4, 661 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b(c x^2)^{3/2}}}{x} + \frac{3 b^{1/3} \sqrt{c x^2} \sqrt{a+b(c x^2)^{3/2}}}{x \left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)} - \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{1/3} b^{1/3} \sqrt{c x^2} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(2 x \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right) + \\
 & \left(\sqrt{2} 3^{3/4} a^{1/3} b^{1/3} \sqrt{c x^2} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(x \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^2} dx$$

Problem 2947: Unable to integrate problem.

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^5} dx$$

Optimal (type 4, 681 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b(c x^2)^{3/2}}}{4 x^4} - \frac{3 b c^2 \sqrt{a+b(c x^2)^{3/2}}}{8 a \sqrt{c x^2}} + \frac{3 b^{4/3} c^2 \sqrt{a+b(c x^2)^{3/2}}}{8 a \left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)} - \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{4/3} c^2 \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \left(16 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right) + \\
 & \left(3^{3/4} b^{4/3} c^2 \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^2 - a^{1/3} b^{1/3} \sqrt{c x^2}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2}} \right], -7-4 \sqrt{3} \right] \right) / \\
 & \left(4 \sqrt{2} a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^2} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^2} \right)^2}} \sqrt{a+b(c x^2)^{3/2}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b(c x^2)^{3/2}}}{x^5} dx$$

Problem 2948: Unable to integrate problem.

$$\int (d x)^m \sqrt{a+b(c x^2)^{3/2}} dx$$

Optimal (type 5, 86 leaves, 3 steps):

$$\begin{aligned}
 & \left((d x)^{1+m} \sqrt{a+b(c x^2)^{3/2}} \text{Hypergeometric2F1} \left[-\frac{1}{2}, \frac{1+m}{3}, \frac{4+m}{3}, -\frac{b(c x^2)^{3/2}}{a} \right] \right) / \\
 & \left(d(1+m) \sqrt{1 + \frac{b(c x^2)^{3/2}}{a}} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int (d x)^m \sqrt{a + b (c x^2)^{3/2}} dx$$

Problem 2951: Unable to integrate problem.

$$\int (d x)^m \sqrt{a + \frac{b}{(c x^2)^{3/2}}} dx$$

Optimal (type 5, 90 leaves, 4 steps):

$$\left((d x)^{1+m} \sqrt{a + \frac{b}{(c x^2)^{3/2}}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{3}(-1-m), \frac{2-m}{3}, -\frac{b}{a(c x^2)^{3/2}}\right] \right) /$$

$$\left(d(1+m) \sqrt{1 + \frac{b}{a(c x^2)^{3/2}}} \right)$$

Result (type 8, 25 leaves):

$$\int (d x)^m \sqrt{a + \frac{b}{(c x^2)^{3/2}}} dx$$

Problem 2952: Unable to integrate problem.

$$\int \frac{1}{1 + (x^3)^{2/3}} dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$\frac{x \operatorname{ArcTan}\left[(x^3)^{1/3}\right]}{(x^3)^{1/3}}$$

Result (type 8, 13 leaves):

$$\int \frac{1}{1 + (x^3)^{2/3}} dx$$

Problem 2955: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^3}}}{x} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{4}{3} \sqrt{a + b \sqrt{c x^3}} - \frac{4}{3} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sqrt{c x^3}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c x^3}}}{x} dx$$

Problem 2956: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c x^3}}}{x^4} dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$-\frac{\sqrt{a+b\sqrt{c x^3}}}{3 x^3} - \frac{b c \sqrt{a+b\sqrt{c x^3}}}{6 a \sqrt{c x^3}} + \frac{b^2 c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c x^3}}}{\sqrt{a}}\right]}{6 a^{3/2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c x^3}}}{x^4} dx$$

Problem 2957: Unable to integrate problem.

$$\int x \sqrt{a+b\sqrt{c x^3}} dx$$

Optimal (type 4, 400 leaves, 5 steps):

$$\frac{4}{11} x^2 \sqrt{a + b \sqrt{c x^3}} + \frac{12 a x^2 \sqrt{a + b \sqrt{c x^3}}}{55 b \sqrt{c x^3}} -$$

$$\left(8 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(55 b^{4/3} c^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \sqrt{a + b \sqrt{c x^3}} \right)$$

Result (type 8, 21 leaves):

$$\int x \sqrt{a + b \sqrt{c x^3}} dx$$

Problem 2958: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^3}}}{x^2} dx$$

Optimal (type 4, 355 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b\sqrt{cx^3}}}{x} + \left(3^{3/4} \sqrt{2+\sqrt{3}} b^{2/3} c^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right) \right. \\
 & \left. \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(\sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)^2}} \sqrt{a+b\sqrt{cx^3}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2} dx$$

Problem 2959: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5} dx$$

Optimal (type 4, 434 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4} + \frac{21b^2c\sqrt{a+b\sqrt{cx^3}}}{160a^2x} - \frac{3bc^3x^5\sqrt{a+b\sqrt{cx^3}}}{40a(c x^3)^{5/2}} + \\
 & \left(7 \times 3^{3/4} \sqrt{2+\sqrt{3}} b^{8/3} c^{4/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(160 a^2 \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)^2}} \sqrt{a+b\sqrt{cx^3}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5} dx$$

Problem 2960: Unable to integrate problem.

$$\int x^3 \sqrt{a+b\sqrt{cx^3}} dx$$

Optimal (type 4, 843 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{120 a^2 x \sqrt{a+b} \sqrt{c x^3}}{1729 b^2 c} + \frac{4}{19} x^4 \sqrt{a+b} \sqrt{c x^3} + \\
 & \frac{12 a x \sqrt{c x^3} \sqrt{a+b} \sqrt{c x^3}}{247 b c} + \frac{480 a^3 \sqrt{a+b} \sqrt{c x^3}}{1729 b^{8/3} c^{4/3} \left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)} - \\
 & \left(240 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{10/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} c^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \sqrt{a+b} \sqrt{c x^3}} \right) + \\
 & \left(160 \sqrt{2} 3^{3/4} a^{10/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{(1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}} \right], -7-4\sqrt{3} \right] \right) / \\
 & \left(1729 b^{8/3} c^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)}{\left((1+\sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \sqrt{a+b} \sqrt{c x^3}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int x^3 \sqrt{a+b} \sqrt{c x^3} \, dx$$

Problem 2961: Unable to integrate problem.

$$\int \sqrt{a + b \sqrt{c x^3}} \, dx$$

Optimal (type 4, 770 leaves, 6 steps):

$$\frac{4}{7} x \sqrt{a + b \sqrt{c x^3}} + \frac{12 a \sqrt{a + b \sqrt{c x^3}}}{7 b^{2/3} c^{1/3} \left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)} -$$

$$\left(6 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 b^{2/3} c^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \sqrt{a + b \sqrt{c x^3}} \right) +$$

$$\left(4 \sqrt{2} 3^{3/4} a^{4/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{(1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(7 b^{2/3} c^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)}{\left((1 + \sqrt{3}) a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}} \right)^2}} \sqrt{a + b \sqrt{c x^3}} \right)$$

Result (type 8, 19 leaves):

$$\int \sqrt{a + b \sqrt{c x^3}} \, dx$$

Problem 2962: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{c x^3}}}{x^3} \, dx$$

Optimal (type 4, 810 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} - \frac{3bcx\sqrt{a+b\sqrt{cx^3}}}{4a\sqrt{cx^3}} + \frac{3b^{4/3}c^{2/3}\sqrt{a+b\sqrt{cx^3}}}{4a\left(\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3}c^{2/3}x^2}{\sqrt{cx^3}}\right)} \\
 & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{4/3} c^{2/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}\right)^2}} \right. \\
 & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(8 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)}{\left(\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}\right)^2}} \sqrt{a+b\sqrt{cx^3}} \right) + \\
 & \left(3^{3/4} b^{4/3} c^{2/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c^{1/3} x - \frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}\right)^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left(2 \sqrt{2} a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}} \right)}{\left(\left(1+\sqrt{3}\right)a^{1/3} + \frac{b^{1/3} c^{2/3} x^2}{\sqrt{cx^3}}\right)^2}} \sqrt{a+b\sqrt{cx^3}} \right)
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3} dx$$

Problem 2963: Unable to integrate problem.

$$\int x^{17} \sqrt{a + b (c x^3)^{3/2}} dx$$

Optimal (type 2, 116 leaves, 4 steps):

$$-\frac{4 a^3 \left(a + b (c x^3)^{3/2}\right)^{3/2}}{27 b^4 c^6} + \frac{4 a^2 \left(a + b (c x^3)^{3/2}\right)^{5/2}}{15 b^4 c^6} - \frac{4 a \left(a + b (c x^3)^{3/2}\right)^{7/2}}{21 b^4 c^6} + \frac{4 \left(a + b (c x^3)^{3/2}\right)^{9/2}}{81 b^4 c^6}$$

Result (type 8, 23 leaves):

$$\int x^{17} \sqrt{a + b (c x^3)^{3/2}} dx$$

Problem 2964: Unable to integrate problem.

$$\int x^8 \sqrt{a + b (c x^3)^{3/2}} dx$$

Optimal (type 2, 56 leaves, 4 steps):

$$-\frac{4 a \left(a + b (c x^3)^{3/2}\right)^{3/2}}{27 b^2 c^3} + \frac{4 \left(a + b (c x^3)^{3/2}\right)^{5/2}}{45 b^2 c^3}$$

Result (type 8, 23 leaves):

$$\int x^8 \sqrt{a + b (c x^3)^{3/2}} dx$$

Problem 2965: Unable to integrate problem.

$$\int \frac{\sqrt{a + b (c x^3)^{3/2}}}{x} dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{4}{9} \sqrt{a + b (c x^3)^{3/2}} - \frac{4}{9} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b (c x^3)^{3/2}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b (c x^3)^{3/2}}}{x} dx$$

Problem 2966: Unable to integrate problem.

$$\int \frac{\sqrt{a + b (c x^3)^{3/2}}}{x^{10}} dx$$

Optimal (type 3, 101 leaves, 6 steps):

$$-\frac{\sqrt{a + b (c x^3)^{3/2}}}{9 x^9} - \frac{b c^3 \sqrt{a + b (c x^3)^{3/2}}}{18 a (c x^3)^{3/2}} + \frac{b^2 c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a + b (c x^3)^{3/2}}}{\sqrt{a}}\right]}{18 a^{3/2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a + b (c x^3)^{3/2}}}{x^{10}} dx$$

Problem 2967: Result unnecessarily involves higher level functions.

$$\int x^2 \sqrt{a + b (c x^3)^{3/2}} dx$$

Optimal (type 4, 642 leaves, 7 steps):

$$\frac{4}{21} x^3 \sqrt{a+b (c x^3)^{3/2}} + \frac{4 a \sqrt{a+b (c x^3)^{3/2}}}{7 b^{2/3} c \left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3} \right)} -$$

$$\left(2 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{4/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^3} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^3 - a^{1/3} b^{1/3} \sqrt{c x^3}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3} \right)^2}} \right.$$

$$\left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3}} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left(7 b^{2/3} c \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^3} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3} \right)^2}} \sqrt{a+b (c x^3)^{3/2}} \right) +$$

$$\left(4 \sqrt{2} a^{4/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^3} \right) \sqrt{\frac{a^{2/3} + b^{2/3} c x^3 - a^{1/3} b^{1/3} \sqrt{c x^3}}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3}}{(1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3}} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left(7 \times 3^{1/4} b^{2/3} c \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} \sqrt{c x^3} \right)}{\left((1+\sqrt{3}) a^{1/3} + b^{1/3} \sqrt{c x^3} \right)^2}} \sqrt{a+b (c x^3)^{3/2}} \right)$$

Result (type 5, 89 leaves):

$$\left(x^3 \left(4 (a+b (c x^3)^{3/2}) + 3 a \sqrt{\frac{a+b (c x^3)^{3/2}}{a}} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{b (c x^3)^{3/2}}{a} \right] \right) \right) /$$

$$\left(21 \sqrt{a+b (c x^3)^{3/2}} \right)$$

Problem 2968: Unable to integrate problem.

$$\int x^9 \sqrt{a+b (c x^3)^{3/2}} dx$$

Optimal (type 5, 170 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{792 a^2 x \sqrt{a+b (c x^3)^{3/2}}}{19747 b^2 c^3} + \frac{4}{49} x^{10} \sqrt{a+b (c x^3)^{3/2}} + \frac{36 a x (c x^3)^{3/2} \sqrt{a+b (c x^3)^{3/2}}}{1519 b c^3} + \\
 & \frac{792 a^3 x \sqrt{1 + \frac{b (c x^3)^{3/2}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{2}{9}, \frac{1}{2}, \frac{11}{9}, -\frac{b (c x^3)^{3/2}}{a}\right]}{19747 b^2 c^3 \sqrt{a+b (c x^3)^{3/2}}}
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int x^9 \sqrt{a+b (c x^3)^{3/2}} dx$$

Problem 2969: Unable to integrate problem.

$$\int \sqrt{a+b (c x^3)^{3/2}} dx$$

Optimal (type 5, 91 leaves, 5 steps):

$$\frac{4}{13} x \sqrt{a+b (c x^3)^{3/2}} + \frac{9 a x \sqrt{1 + \frac{b (c x^3)^{3/2}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{2}{9}, \frac{1}{2}, \frac{11}{9}, -\frac{b (c x^3)^{3/2}}{a}\right]}{13 \sqrt{a+b (c x^3)^{3/2}}}$$

Result (type 8, 19 leaves):

$$\int \sqrt{a+b (c x^3)^{3/2}} dx$$

Problem 2970: Unable to integrate problem.

$$\int \frac{\sqrt{a+b (c x^3)^{3/2}}}{x^9} dx$$

Optimal (type 5, 139 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+b (c x^3)^{3/2}}}{8 x^8} - \frac{9 b c^3 x \sqrt{a+b (c x^3)^{3/2}}}{112 a (c x^3)^{3/2}} - \\
 & \frac{45 b^2 c^3 x \sqrt{1 + \frac{b (c x^3)^{3/2}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{2}{9}, \frac{1}{2}, \frac{11}{9}, -\frac{b (c x^3)^{3/2}}{a}\right]}{448 a \sqrt{a+b (c x^3)^{3/2}}}
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b (c x^3)^{3/2}}}{x^9} dx$$

Problem 2971: Unable to integrate problem.

$$\int (d x)^m \sqrt{a + b (c x^3)^{3/2}} dx$$

Optimal (type 5, 84 leaves, 5 steps):

$$\left(x (d x)^m \sqrt{a + b (c x^3)^{3/2}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2(1+m)}{9}, 1 + \frac{2(1+m)}{9}, -\frac{b(c x^3)^{3/2}}{a}\right] \right) /$$

$$\left((1+m) \sqrt{1 + \frac{b(c x^3)^{3/2}}{a}} \right)$$

Result (type 8, 25 leaves):

$$\int (d x)^m \sqrt{a + b (c x^3)^{3/2}} dx$$

Problem 2974: Unable to integrate problem.

$$\int (d x)^m \sqrt{a + \frac{b}{(c x^3)^{3/2}}} dx$$

Optimal (type 5, 102 leaves, 6 steps):

$$\left(x (d x)^m \sqrt{a + \frac{b c^3 x^9}{(c x^3)^{9/2}}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2}{9}(1+m), \frac{1}{9}(7-2m), -\frac{b c^3 x^9}{a (c x^3)^{9/2}}\right] \right) /$$

$$\left((1+m) \sqrt{1 + \frac{b c^3 x^9}{a (c x^3)^{9/2}}} \right)$$

Result (type 8, 25 leaves):

$$\int (d x)^m \sqrt{a + \frac{b}{(c x^3)^{3/2}}} dx$$

Problem 2988: Unable to integrate problem.

$$\int \sqrt{a + b \left(\frac{c}{x}\right)^{3/2}} (d x)^m dx$$

Optimal (type 5, 102 leaves, 6 steps):

$$\left(\sqrt{a + \frac{b c^3}{\left(\frac{c}{x}\right)^{3/2} x^3}} x (d x)^m \text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2}{3}(1+m), \frac{1}{3}(1-2m), -\frac{b c^3}{a \left(\frac{c}{x}\right)^{3/2} x^3}\right] \right) /$$

$$\left((1+m) \sqrt{1 + \frac{b c^3}{a \left(\frac{c}{x}\right)^{3/2} x^3}} \right)$$

Result (type 8, 25 leaves):

$$\int \sqrt{a + b \left(\frac{c}{x}\right)^{3/2}} (d x)^m dx$$

Problem 2991: Unable to integrate problem.

$$\int \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{3/2}}} (d x)^m dx$$

Optimal (type 5, 102 leaves, 5 steps):

$$\left(x (d x)^m \sqrt{a + \frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{c^3}} \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{2(1+m)}{3}, \frac{1}{3}(5+2m), -\frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{a c^3}\right] \right) /$$

$$\left((1+m) \sqrt{1 + \frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{a c^3}} \right)$$

Result (type 8, 25 leaves):

$$\int \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{3/2}}} (d x)^m dx$$

Problem 2992: Unable to integrate problem.

$$\int \frac{(d x)^m}{\sqrt{a + b \left(\frac{c}{x}\right)^{3/2}}} dx$$

Optimal (type 5, 102 leaves, 6 steps):

$$\left(\sqrt{1 + \frac{b c^3}{a \left(\frac{c}{x}\right)^{3/2} x^3}} x (d x)^m \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2}{3}(1+m), \frac{1}{3}(1-2m), -\frac{b c^3}{a \left(\frac{c}{x}\right)^{3/2} x^3}\right] \right) /$$

$$\left((1+m) \sqrt{a + \frac{b c^3}{\left(\frac{c}{x}\right)^{3/2} x^3}} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{(d x)^m}{\sqrt{a + b \left(\frac{c}{x}\right)^{3/2}}} dx$$

Problem 2995: Unable to integrate problem.

$$\int \frac{(d x)^m}{\sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} dx$$

Optimal (type 5, 102 leaves, 5 steps):

$$\left(x (d x)^m \sqrt{1 + \frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{a c^3}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2(1+m)}{3}, \frac{1}{3}(5+2m), -\frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{a c^3}\right] \right) /$$

$$\left((1+m) \sqrt{a + \frac{b \left(\frac{c}{x}\right)^{3/2} x^3}{c^3}} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{(d x)^m}{\sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} dx$$

Problem 2999: Unable to integrate problem.

$$\int \frac{x^3}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Optimal (type 3, 101 leaves, 3 steps):

$$\frac{a^2 x^4 (c x^n)^{-3/n}}{b^3} - \frac{a x^4 (c x^n)^{-2/n}}{2 b^2} + \frac{x^4 (c x^n)^{-1/n}}{3 b} - \frac{a^3 x^4 (c x^n)^{-4/n} \text{Log}\left[a + b (c x^n)^{\frac{1}{n}}\right]}{b^4}$$

Result (type 8, 21 leaves):

$$\int \frac{x^3}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Problem 3000: Unable to integrate problem.

$$\int \frac{x^2}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{a x^3 (c x^n)^{-2/n}}{b^2} + \frac{x^3 (c x^n)^{-1/n}}{2 b} + \frac{a^2 x^3 (c x^n)^{-3/n} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{b^3}$$

Result (type 8, 21 leaves):

$$\int \frac{x^2}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Problem 3001: Unable to integrate problem.

$$\int \frac{x}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{x^2 (c x^n)^{-1/n}}{b} - \frac{a x^2 (c x^n)^{-2/n} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{b^2}$$

Result (type 8, 19 leaves):

$$\int \frac{x}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Problem 3004: Unable to integrate problem.

$$\int \frac{1}{x^2 (a + b (c x^n)^{\frac{1}{n}})} dx$$

Optimal (type 3, 60 leaves, 3 steps):

$$-\frac{1}{a x} - \frac{b (c x^n)^{\frac{1}{n}} \operatorname{Log}[x]}{a^2 x} + \frac{b (c x^n)^{\frac{1}{n}} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{a^2 x}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^2 \left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)} dx$$

Problem 3005: Unable to integrate problem.

$$\int \frac{1}{x^3 \left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)} dx$$

Optimal (type 3, 87 leaves, 3 steps):

$$-\frac{1}{2 a x^2} + \frac{b \left(c x^n \right)^{\frac{1}{n}}}{a^2 x^2} + \frac{b^2 \left(c x^n \right)^{2/n} \operatorname{Log}[x]}{a^3 x^2} - \frac{b^2 \left(c x^n \right)^{2/n} \operatorname{Log}\left[a + b \left(c x^n \right)^{\frac{1}{n}} \right]}{a^3 x^2}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^3 \left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)} dx$$

Problem 3006: Unable to integrate problem.

$$\int \frac{x^3}{\left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)^2} dx$$

Optimal (type 3, 114 leaves, 3 steps):

$$-\frac{2 a x^4 \left(c x^n \right)^{-3/n}}{b^3} + \frac{x^4 \left(c x^n \right)^{-2/n}}{2 b^2} + \frac{a^3 x^4 \left(c x^n \right)^{-4/n}}{b^4 \left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)} + \frac{3 a^2 x^4 \left(c x^n \right)^{-4/n} \operatorname{Log}\left[a + b \left(c x^n \right)^{\frac{1}{n}} \right]}{b^4}$$

Result (type 8, 21 leaves):

$$\int \frac{x^3}{\left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)^2} dx$$

Problem 3007: Unable to integrate problem.

$$\int \frac{x^2}{\left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)^2} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$\frac{x^3 \left(c x^n \right)^{-2/n}}{b^2} - \frac{a^2 x^3 \left(c x^n \right)^{-3/n}}{b^3 \left(a + b \left(c x^n \right)^{\frac{1}{n}} \right)} - \frac{2 a x^3 \left(c x^n \right)^{-3/n} \operatorname{Log}\left[a + b \left(c x^n \right)^{\frac{1}{n}} \right]}{b^3}$$

Result (type 8, 21 leaves):

$$\int \frac{x^2}{\left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Problem 3008: Unable to integrate problem.

$$\int \frac{x}{\left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$\frac{a x^2 \left(c x^n\right)^{-2/n} + x^2 \left(c x^n\right)^{-2/n} \operatorname{Log}\left[a + b \left(c x^n\right)^{\frac{1}{n}}\right]}{b^2 \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2}$$

Result (type 8, 19 leaves):

$$\int \frac{x}{\left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Problem 3011: Unable to integrate problem.

$$\int \frac{1}{x^2 \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Optimal (type 3, 94 leaves, 3 steps):

$$-\frac{1}{a^2 x} - \frac{b \left(c x^n\right)^{\frac{1}{n}}}{a^2 x \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)} - \frac{2 b \left(c x^n\right)^{\frac{1}{n}} \operatorname{Log}[x]}{a^3 x} + \frac{2 b \left(c x^n\right)^{\frac{1}{n}} \operatorname{Log}\left[a + b \left(c x^n\right)^{\frac{1}{n}}\right]}{a^3 x}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^2 \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Problem 3012: Unable to integrate problem.

$$\int \frac{1}{x^3 \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2} dx$$

Optimal (type 3, 125 leaves, 3 steps):

$$-\frac{1}{2 a^2 x^2} + \frac{2 b (c x^n)^{\frac{1}{n}}}{a^3 x^2} + \frac{b^2 (c x^n)^{2/n}}{a^3 x^2 (a + b (c x^n)^{\frac{1}{n}})} + \frac{3 b^2 (c x^n)^{2/n} \operatorname{Log}[x]}{a^4 x^2} - \frac{3 b^2 (c x^n)^{2/n} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{a^4 x^2}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^3 (a + b (c x^n)^{\frac{1}{n}})^2} dx$$

Problem 3014: Unable to integrate problem.

$$\int \frac{x}{(1 + (x^n)^{\frac{1}{n}})^2} dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{x^2 (x^n)^{-2/n}}{1 + (x^n)^{\frac{1}{n}}} + x^2 (x^n)^{-2/n} \operatorname{Log}[1 + (x^n)^{\frac{1}{n}}]$$

Result (type 8, 15 leaves):

$$\int \frac{x}{(1 + (x^n)^{\frac{1}{n}})^2} dx$$

Problem 3024: Unable to integrate problem.

$$\int \frac{1}{a + b (c x^n)^{2/n}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{x (c x^n)^{-1/n} \operatorname{ArcTan}\left[\frac{\sqrt{b} (c x^n)^{\frac{1}{n}}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{a + b (c x^n)^{2/n}} dx$$

Problem 3025: Unable to integrate problem.

$$\int \frac{1}{(a + b (c x^n)^{2/n})^2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{x}{2 a (a+b (c x^n)^{2/n})} + \frac{x (c x^n)^{-1/n} \text{ArcTan}\left[\frac{\sqrt{b} (c x^n)^{\frac{1}{n}}}{\sqrt{a}}\right]}{2 a^{3/2} \sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(a+b (c x^n)^{2/n})^2} dx$$

Problem 3026: Unable to integrate problem.

$$\int \frac{1}{(a+b (c x^n)^{2/n})^3} dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{x}{4 a (a+b (c x^n)^{2/n})^2} + \frac{3 x}{8 a^2 (a+b (c x^n)^{2/n})} + \frac{3 x (c x^n)^{-1/n} \text{ArcTan}\left[\frac{\sqrt{b} (c x^n)^{\frac{1}{n}}}{\sqrt{a}}\right]}{8 a^{5/2} \sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(a+b (c x^n)^{2/n})^3} dx$$

Problem 3027: Unable to integrate problem.

$$\int \frac{1}{1+4 \sqrt{x^4}} dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x \text{ArcTan}\left[2 (x^4)^{1/4}\right]}{2 (x^4)^{1/4}}$$

Result (type 8, 15 leaves):

$$\int \frac{1}{1+4 \sqrt{x^4}} dx$$

Problem 3028: Unable to integrate problem.

$$\int \frac{1}{1-4 \sqrt{x^4}} dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x \operatorname{ArcTanh}\left[2\left(x^4\right)^{1/4}\right]}{2\left(x^4\right)^{1/4}}$$

Result (type 8, 15 leaves):

$$\int \frac{1}{1-4\sqrt{x^4}} dx$$

Problem 3029: Unable to integrate problem.

$$\int \frac{1}{1+4\left(x^6\right)^{1/3}} dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x \operatorname{ArcTan}\left[2\left(x^6\right)^{1/6}\right]}{2\left(x^6\right)^{1/6}}$$

Result (type 9, 142 leaves):

$$\frac{1}{24\left(-x^6\right)^{5/6}}\left(-2x\left(-x^{12}\right)^{1/3} \operatorname{Beta}\left[-64x^6, \frac{1}{2}, 0\right]+2x\left(x^6\right)^{2/3} \operatorname{Beta}\left[-64x^6, \frac{5}{6}, 0\right]+\left(-x^6\right)^{5/6}\left(-2 \operatorname{ArcTan}\left[\sqrt{3}-4x\right]+4 \operatorname{ArcTan}\left[2x\right]+2 \operatorname{ArcTan}\left[\sqrt{3}+4x\right]-\sqrt{3} \operatorname{Log}\left[1-2\sqrt{3}x+4x^2\right]+\sqrt{3} \operatorname{Log}\left[1+2\sqrt{3}x+4x^2\right]\right)\right)$$

Problem 3030: Unable to integrate problem.

$$\int \frac{1}{1-4\left(x^6\right)^{1/3}} dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x \operatorname{ArcTanh}\left[2\left(x^6\right)^{1/6}\right]}{2\left(x^6\right)^{1/6}}$$

Result (type 9, 123 leaves):

$$\frac{1}{24}\left(2\sqrt{3} \operatorname{ArcTan}\left[\frac{-1+4x}{\sqrt{3}}\right]+2\sqrt{3} \operatorname{ArcTan}\left[\frac{1+4x}{\sqrt{3}}\right]+\frac{2x \operatorname{Beta}\left[64x^6, \frac{1}{2}, 0\right]}{\left(x^6\right)^{1/6}}+\frac{2x \operatorname{Beta}\left[64x^6, \frac{5}{6}, 0\right]}{\left(x^6\right)^{1/6}}-2 \operatorname{Log}\left[1-2x\right]+2 \operatorname{Log}\left[1+2x\right]-\operatorname{Log}\left[1-2x+4x^2\right]+\operatorname{Log}\left[1+2x+4x^2\right]\right)$$

Problem 3031: Unable to integrate problem.

$$\int \frac{1}{1 + 4 (x^{2n})^{\frac{1}{n}}} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{1}{2} x (x^{2n})^{-\frac{1}{2}/n} \text{ArcTan} [2 (x^{2n})^{\frac{1}{2}/n}]$$

Result (type 8, 17 leaves):

$$\int \frac{1}{1 + 4 (x^{2n})^{\frac{1}{n}}} dx$$

Problem 3032: Unable to integrate problem.

$$\int \frac{1}{1 - 4 (x^{2n})^{\frac{1}{n}}} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{1}{2} x (x^{2n})^{-\frac{1}{2}/n} \text{ArcTanh} [2 (x^{2n})^{\frac{1}{2}/n}]$$

Result (type 8, 17 leaves):

$$\int \frac{1}{1 - 4 (x^{2n})^{\frac{1}{n}}} dx$$

Problem 3036: Unable to integrate problem.

$$\int \frac{1}{a + b (c x^n)^{\frac{1}{3}}} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$-\frac{x (c x^n)^{-1/n} \text{ArcTan} \left[\frac{a^{1/3} - 2 b^{1/3} (c x^n)^{\frac{1}{n}}}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{2/3} b^{1/3}} + \frac{x (c x^n)^{-1/n} \text{Log} [a^{1/3} + b^{1/3} (c x^n)^{\frac{1}{n}}]}{3 a^{2/3} b^{1/3}}$$

$$-\frac{x (c x^n)^{-1/n} \text{Log} [a^{2/3} - a^{1/3} b^{1/3} (c x^n)^{\frac{1}{n}} + b^{2/3} (c x^n)^{2/n}]}{6 a^{2/3} b^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{a + b (c x^n)^{\frac{1}{3}}} dx$$

Problem 3037: Unable to integrate problem.

$$\int \frac{1}{(a + b (c x^n)^{3/n})^2} dx$$

Optimal (type 3, 210 leaves, 8 steps):

$$\frac{x}{3 a (a + b (c x^n)^{3/n})} - \frac{2 x (c x^n)^{-1/n} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c x^n)^{1/n}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} b^{1/3}} +$$

$$\frac{2 x (c x^n)^{-1/n} \operatorname{Log}\left[a^{1/3} + b^{1/3} (c x^n)^{1/n}\right]}{9 a^{5/3} b^{1/3}} - \frac{x (c x^n)^{-1/n} \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c x^n)^{1/n} + b^{2/3} (c x^n)^{2/n}\right]}{9 a^{5/3} b^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(a + b (c x^n)^{3/n})^2} dx$$

Problem 3038: Unable to integrate problem.

$$\int \frac{1}{(a + b (c x^n)^{3/n})^3} dx$$

Optimal (type 3, 235 leaves, 9 steps):

$$\frac{x}{6 a (a + b (c x^n)^{3/n})^2} + \frac{5 x}{18 a^2 (a + b (c x^n)^{3/n})} - \frac{5 x (c x^n)^{-1/n} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c x^n)^{1/n}}{\sqrt{3} a^{1/3}}\right]}{9 \sqrt{3} a^{8/3} b^{1/3}} +$$

$$\frac{5 x (c x^n)^{-1/n} \operatorname{Log}\left[a^{1/3} + b^{1/3} (c x^n)^{1/n}\right]}{27 a^{8/3} b^{1/3}} - \frac{5 x (c x^n)^{-1/n} \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c x^n)^{1/n} + b^{2/3} (c x^n)^{2/n}\right]}{54 a^{8/3} b^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(a + b (c x^n)^{3/n})^3} dx$$

Problem 3045: Unable to integrate problem.

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} x^m dx$$

Optimal (type 6, 230 leaves, 4 steps):

$$\left(\sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}} x^{1+m} \operatorname{AppellF1}\left[-2(1+m), -\frac{1}{2}, -\frac{1}{2}, -1-2m, -\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}(b\sqrt{d}-\sqrt{-4ac+b^2d})}, -\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}(b\sqrt{d}+\sqrt{-4ac+b^2d})}\right] \right) \sqrt{\left(1+m\right) \sqrt{1 + \frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}(b\sqrt{d}-\sqrt{-4ac+b^2d})}} \sqrt{1 + \frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}(b\sqrt{d}+\sqrt{-4ac+b^2d})}}}$$

Result (type 8, 28 leaves):

$$\int \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}} x^m dx$$

Problem 3046: Unable to integrate problem.

$$\int \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}} x^2 dx$$

Optimal (type 3, 333 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{3 b d^3 \left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2}}{10 a^2 \left(\frac{d}{x} \right)^{5/2}} + \frac{7 b d^2 (28 a c - 15 b^2 d) \left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2}}{480 a^4 \left(\frac{d}{x} \right)^{3/2}} + \\
 & \frac{(16 a^2 c^2 - 56 a b^2 c d + 21 b^4 d^2) \left(2 a + b \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}} x}}{256 a^5} - \\
 & \frac{(20 a c - 21 b^2 d) \left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2} x^2}{80 a^3} + \frac{\left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2} x^3}{3 a} + \frac{1}{512 a^{11/2}} \\
 & (4 a c - b^2 d) (16 a^2 c^2 - 56 a b^2 c d + 21 b^4 d^2) \operatorname{ArcTanh} \left[\frac{2 a + b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} \right]
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} x^2 dx$$

Problem 3047: Unable to integrate problem.

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} x dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{5 b d^2 \left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2}}{12 a^2 \left(\frac{d}{x} \right)^{3/2}} - \frac{(4 a c - 5 b^2 d) \left(2 a + b \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}} x}}{32 a^3} + \\
 & \frac{\left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2} x^2}{2 a} - \frac{(4 a c - 5 b^2 d) (4 a c - b^2 d) \operatorname{ArcTanh} \left[\frac{2 a + b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} \right]}{64 a^{7/2}}
 \end{aligned}$$

Result (type 8, 26 leaves):

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} x \, dx$$

Problem 3048: Unable to integrate problem.

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} \, dx$$

Optimal (type 3, 113 leaves, 5 steps):

$$\frac{\left(2 a + b \sqrt{\frac{d}{x}}\right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} x}{2 a} + \frac{(4 a c - b^2 d) \operatorname{ArcTanh}\left[\frac{2 a + b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right]}{4 a^{3/2}}$$

Result (type 8, 24 leaves):

$$\int \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} \, dx$$

Problem 3049: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x} \, dx$$

Optimal (type 3, 145 leaves, 8 steps):

$$-2 \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} + 2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{2 a + b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right] - \frac{b \sqrt{d} \operatorname{ArcTanh}\left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right]}{\sqrt{c}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x} \, dx$$

Problem 3050: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x^2} dx$$

Optimal (type 3, 155 leaves, 6 steps):

$$\frac{b \left(b d + 2 c \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{4 c^2} - \frac{2 \left(a + b \sqrt{\frac{d}{x} + \frac{c}{x}} \right)^{3/2}}{3 c} + \frac{b \sqrt{d} (4 a c - b^2 d) \text{ArcTanh} \left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} \right]}{8 c^{5/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x^2} dx$$

Problem 3051: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x^3} dx$$

Optimal (type 3, 233 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{b (12 a c - 7 b^2 d) \left(b d + 2 c \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}}{64 c^4} + \\
 & \frac{\left(32 a c - 35 b^2 d + 42 b c \sqrt{\frac{d}{x}} \right) \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2}}{120 c^3} - \frac{2 \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2}}{5 c x} - \\
 & \frac{b \sqrt{d} (12 a c - 7 b^2 d) (4 a c - b^2 d) \operatorname{ArcTanh} \left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}} \right]}{128 c^{9/2}}
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^3} dx$$

Problem 3052: Unable to integrate problem.

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4} dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$\begin{aligned}
 & \frac{b (80 a^2 c^2 - 120 a b^2 c d + 33 b^4 d^2) \left(b d + 2 c \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}}{512 c^6} - \frac{1}{6720 c^5} \\
 & \left(1024 a^2 c^2 - 3276 a b^2 c d + 1155 b^4 d^2 + 18 b c (148 a c - 77 b^2 d) \sqrt{\frac{d}{x}} \right) \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2} + \\
 & \frac{11 b \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2} \left(\frac{d}{x} \right)^{3/2}}{42 c^2 d} - \frac{2 \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2}}{7 c x^2} + \frac{(32 a c - 33 b^2 d) \left(a + b \sqrt{\frac{d}{x}} + \frac{c}{x} \right)^{3/2}}{140 c^3 x} + \\
 & \frac{1}{1024 c^{13/2}} b \sqrt{d} (4 a c - b^2 d) (80 a^2 c^2 - 120 a b^2 c d + 33 b^4 d^2) \operatorname{ArcTanh} \left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}} \right]
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{x^4} dx$$

Problem 3053: Unable to integrate problem.

$$\int \frac{x^m}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Optimal (type 6, 230 leaves, 4 steps):

$$\left(\sqrt{1 + \frac{2c \sqrt{\frac{d}{x}}}{\sqrt{d} (b \sqrt{d} - \sqrt{-4ac + b^2 d})}} \sqrt{1 + \frac{2c \sqrt{\frac{d}{x}}}{\sqrt{d} (b \sqrt{d} + \sqrt{-4ac + b^2 d})}} \right. \\ \left. x^{1+m} \operatorname{AppellF1} \left[-2(1+m), \frac{1}{2}, \frac{1}{2}, -1-2m, -\frac{2c \sqrt{\frac{d}{x}}}{\sqrt{d} (b \sqrt{d} - \sqrt{-4ac + b^2 d})}, \right. \right. \\ \left. \left. -\frac{2c \sqrt{\frac{d}{x}}}{\sqrt{d} (b \sqrt{d} + \sqrt{-4ac + b^2 d})} \right] \right) / \left((1+m) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} \right)$$

Result (type 8, 28 leaves):

$$\int \frac{x^m}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Problem 3054: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Optimal (type 3, 386 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{11 b d^3 \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{30 a^2 \left(\frac{d}{x}\right)^{5/2}} + \frac{b d^2 (156 a c - 77 b^2 d) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{160 a^4 \left(\frac{d}{x}\right)^{3/2}} - \\
 & \frac{7 b d (528 a^2 c^2 - 680 a b^2 c d + 165 b^4 d^2) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{1280 a^6 \sqrt{\frac{d}{x}}} + \\
 & \frac{(400 a^2 c^2 - 1176 a b^2 c d + 385 b^4 d^2) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{640 a^5} - \\
 & \frac{(100 a c - 99 b^2 d) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{240 a^3} + \frac{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{3 a} - \frac{1}{512 a^{13/2}} \\
 & (320 a^3 c^3 - 1680 a^2 b^2 c^2 d + 1260 a b^4 c d^2 - 231 b^6 d^3) \operatorname{ArcTanh} \left[\frac{2 a + b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} \right]
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \frac{x^2}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Problem 3055: Unable to integrate problem.

$$\int \frac{x}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Optimal (type 3, 248 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{7 b d^2 \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{12 a^2 \left(\frac{d}{x}\right)^{3/2}} + \frac{5 b d (44 a c - 21 b^2 d) \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{96 a^4 \sqrt{\frac{d}{x}}} \\
 & \frac{(36 a c - 35 b^2 d) \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}} x}{48 a^3} + \frac{\sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}} x^2}{2 a} + \\
 & \frac{(48 a^2 c^2 - 120 a b^2 c d + 35 b^4 d^2) \operatorname{ArcTanh}\left[\frac{2 a+b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right]}{64 a^{9/2}}
 \end{aligned}$$

Result (type 8, 26 leaves):

$$\int \frac{x}{\sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Problem 3056: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{3 b d \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{2 a^2 \sqrt{\frac{d}{x}}} + \frac{\sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}} x}{a} - \frac{(4 a c - 3 b^2 d) \operatorname{ArcTanh}\left[\frac{2 a+b \sqrt{\frac{d}{x}}}{2 \sqrt{a} \sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right]}{4 a^{5/2}}
 \end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{1}{\sqrt{a+b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Problem 3057: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}x}} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right]}{\sqrt{a}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}x}} dx$$

Problem 3058: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}x^2}} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}{c} + \frac{b\sqrt{d} \operatorname{ArcTanh}\left[\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right]}{c^{3/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}x^2}} dx$$

Problem 3059: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}} x^3}} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{\left(16 a c - 15 b^2 d + 10 b c \sqrt{\frac{d}{x}}\right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{12 c^3} - \frac{2 \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{3 c x} - \frac{b \sqrt{d} (12 a c - 5 b^2 d) \operatorname{ArcTanh}\left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}\right]}{8 c^{7/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}} x^3}} dx$$

Problem 3060: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}} x^4}} dx$$

Optimal (type 3, 289 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{1}{960 c^5} \left(1024 a^2 c^2 - 2940 a b^2 c d + 945 b^4 d^2 + 14 b c (92 a c - 45 b^2 d) \sqrt{\frac{d}{x}} \right) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} + \\
 & \frac{9 b \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}} \left(\frac{d}{x}\right)^{3/2}}{20 c^2 d} - \frac{2 \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{5 c x^2} + \frac{(64 a c - 63 b^2 d) \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}}{120 c^3 x} + \\
 & \frac{b \sqrt{d} (240 a^2 c^2 - 280 a b^2 c d + 63 b^4 d^2) \operatorname{ArcTanh} \left[\frac{b d + 2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} \right]}{128 c^{11/2}}
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a + b \sqrt{\frac{d}{x} + \frac{c}{x}}}} dx$$

Problem 3061: Unable to integrate problem.

$$\int \sqrt{\sqrt{\frac{1}{x} + \frac{1}{x}}} dx$$

Optimal (type 2, 26 leaves, 2 steps):

$$\frac{4 \left(\sqrt{\frac{1}{x} + \frac{1}{x}} \right)^{3/2}}{3 \left(\frac{1}{x} \right)^{3/2}}$$

Result (type 8, 17 leaves):

$$\int \sqrt{\sqrt{\frac{1}{x} + \frac{1}{x}}} dx$$

Problem 3062: Unable to integrate problem.

$$\int \sqrt{2 + \sqrt{\frac{1}{x} + \frac{1}{x}}} dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$\frac{1}{4} \left(4 + \sqrt{\frac{1}{x}} \right) \sqrt{2 + \sqrt{\frac{1}{x}} + \frac{1}{x}} x + \frac{7 \operatorname{ArcTanh} \left[\frac{4 + \sqrt{\frac{1}{x}}}{2\sqrt{2} \sqrt{2 + \sqrt{\frac{1}{x}} + \frac{1}{x}}} \right]}{8\sqrt{2}}$$

Result (type 8, 18 leaves):

$$\int \sqrt{2 + \sqrt{\frac{1}{x}} + \frac{1}{x}} dx$$

Problem 3067: Unable to integrate problem.

$$\int \frac{(c x^n)^{\frac{1}{n}}}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$\frac{x}{b} - \frac{a x (c x^n)^{-1/n} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{b^2}$$

Result (type 8, 27 leaves):

$$\int \frac{(c x^n)^{\frac{1}{n}}}{a + b (c x^n)^{\frac{1}{n}}} dx$$

Problem 3068: Unable to integrate problem.

$$\int \frac{(c x^n)^{\frac{1}{n}}}{(a + b (c x^n)^{\frac{1}{n}})^2} dx$$

Optimal (type 3, 63 leaves, 4 steps):

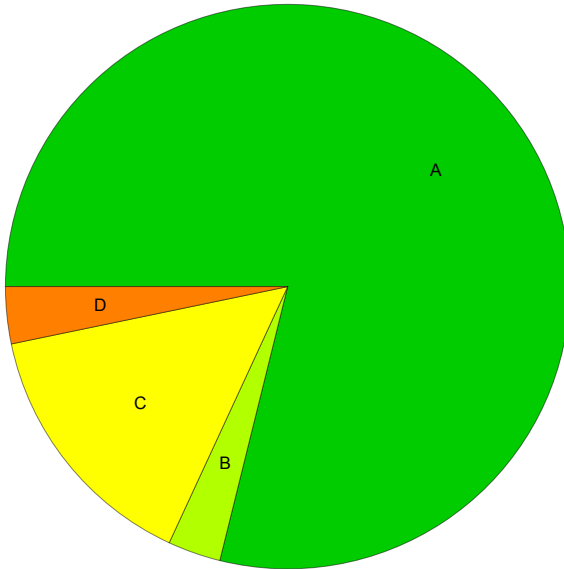
$$\frac{a x (c x^n)^{-1/n}}{b^2 (a + b (c x^n)^{\frac{1}{n}})} + \frac{x (c x^n)^{-1/n} \operatorname{Log}[a + b (c x^n)^{\frac{1}{n}}]}{b^2}$$

Result (type 8, 27 leaves):

$$\int \frac{(c x^n)^{\frac{1}{n}}}{(a + b (c x^n)^{\frac{1}{n}})^2} dx$$

Summary of Integration Test Results

3071 integration problems



- A - 2422 optimal antiderivatives
- B - 93 more than twice size of optimal antiderivatives
- C - 456 unnecessarily complex antiderivatives
- D - 100 unable to integrate problems
- E - 0 integration timeouts